ANALYSIS AND MODIFICATION OF LINEAR CORRELATORS FOR IMAGE PATTERN CLASSIFICATION

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ABSTRACT

This paper considers linear correlation filters used for image pattern recognition. First, we develop a statistical theory to predict the classification performance of a general class of correlation filters for wide sense stationary (WSS) clutter. This analysis includes as special cases the Synthetic Discriminant Function (SDF), the Minimum Variance SDF (MVSDF), and the Minimum Average Correlation Energy (MACE) filters. Second, we develop a modified filter design applicable to nonzero mean noise; this latter case occurs in many applications where the magnitude image is used for classification. We compare the performance of several filters on synthetic radar imagery.

1. INTRODUCTION

Linear correlators are widely used for pattern recognition of optical and synthetic aperture radar (SAR) images. Popular correlator filters include the SDF, the MVSDF, and the MACE filter and their variations [1, 2], which are referred to as SDF-type correlator filters. SDF-type correlators are motivated by matched filters found in communication systems. One correlator is built per class of objects for classification, and classification is based on threshold tests of the correlator outputs. Different SDF-type filters can be derived under different optimization criteria.

Previous performance evaluations of SDF-type correlators employed Monte Carlo simulation studies to consider inter-class performance and detection rates for additive zero-mean white noise [3, 4]. In this paper we first provide a statistical analysis of SDF-type correlator performance for correlated WSS clutter. When the noise is additive Gaussian, this analysis yields analytical expressions for the correct classification probabilities for these correlators.

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In addition, we develop a class of correlators applicable to images with nonzero-mean noise. Such noise arises, for example, in synthetic aperture radar (SAR) applications, where the noise is additive and zero mean in a complex SAR image, but the correlation filters are applied to the *magnitude* of that image. The standard correlation filters are based on zero mean noise and can give poor performance when the noise is nonzero mean. We develop modifications for SDF-type correlators for SAR image recognition applications, and demonstrate improved performance of the modified filters through simulation studies.

2. SDF-TYPE CORRELATORS ON IMAGE CLASSIFICATION APPLICATIONS

We are given a set of n_i training vectors $\{v_{ij}\}_{j=1}^{n_i}$ for each i in one of the I classes; each vector is formed by stacking pixel values from a two-dimensional image. The processing goal is to classify a distorted data vector $\mathbf{y}$ into one of I classes. The "distortion" is caused by both noise and by deviations such as viewing the object at an angle not present in the training images.

SDF-type correlators are often used to classify a measured vector $\mathbf{y}$. One designs correlation filters h_i, one filter per class, and assigns $\mathbf{y}$ to the class whose correlation output is maximum. Each correlator h_i or its corresponding two-dimensional discrete Fourier transform, H_i, is designed such that:

1. h_i is linear circularly shift invariant.
2. h_i minimizes $h_i^* A h_i$ or $H_i^* A H_i$ for some user-specified positive semidefinite matrix A.
3. The output at the origin for filter h_i is 1 for input training vectors from the ith class and 0 for input vectors from all other classes. That is, $v_{kj}^* h_i = \delta_{i-k}$, for all $k = 1, \ldots, I$, and all $j = 1, \ldots, n_k$.

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It is well-known [1] that the solution to the above design constraints is given by one of

$$h_i = A^{-1}x(x^*A^{-1}x)^{-1}u_i \quad (1)$$
$$H_i = d \cdot A^{-1}X(X^*A^{-1}X)^{-1}u_i \quad (2)$$

depending on whether the second constraint is imposed in the spatial or frequency domain. Here, x is a matrix of training vectors, X is its corresponding discrete Fourier transform, d is a constant, and each element of u_i is a 0 or 1 corresponding to the third constraint above. Particular choices of domain and A result in some common filters. Table 1 lists the constraints and minimization criteria for the SDF, MVSDF, and MACE filters. In Table 1, $R_{\mathbf{nn}}$ is the noise covariance matrix, and D is a diagonal matrix whose diagonal elements are the average magnitude spectra of the training patterns.

Table 1: The SDF-type correlators

Correlator	Constraint	Criterion to minimize:
SDF	$x^*h = u$	correlator energy: h^*h
MVSDF	$x^*h = u$	correlator output variance at the origin: $h^*R_{\mathbf{nn}}h$
MACE	$X^*H = d \cdot u$	avg. corr. energy: H^*DH

3. THEORETICAL PERFORMANCE

In this section we derive the theoretical detection rates of SDF-type correlators. To simplify the notation, we assume that there are four classes of targets, $\{\omega_i\}_{i=1}^4$, and that four correlators are synthesized; the generalization to more than four classes is obvious.

Let the test pattern be $\mathbf{y} = f(v_{ij}, \mathbf{n})$, where v_{ij} is one of the training images from the ith class, $\mathbf{n}$ is the noise process, and f is a real-valued function. Let the four correlators be $\{h_k\}_{k=1}^4$. The four correlator outputs at the origin are $\mathbf{z}_k = h_k^*\mathbf{y}$. The decision yields the lth class if $\mathbf{z}_l > \mathbf{z}_k$, for all $k \neq l$. The correct detection rate for the given correlator h, and the given noise model is:

$$Pc = \sum_{i=1}^{4}\sum_{j=1}^{n_i} P_{ij}Pc_{ij} \quad (3)$$

Where P_{ij} is the prior probability,

$$Pc_{1j} = Prob(\mathbf{z}_1 > \mathbf{z}_2, \mathbf{z}_1 > \mathbf{z}_3, \mathbf{z}_1 > \mathbf{z}_4 | v_{1j})$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{z_1}\int_{-\infty}^{z_1}\int_{-\infty}^{z_1} f_{\mathbf{z}|v_{1j}}(z_1, z_2, z_3, z_4)dz_2dz_3dz_4dz_1, \quad (4)$$

and so forth for Pc_{2j}, Pc_{3j} and Pc_{4j}.

Case I: Additive Gaussian Noise
If $\mathbf{n}$ is additive Gaussian then $\mathbf{z} = [\mathbf{z}_1\ \mathbf{z}_2\ \mathbf{z}_3\ \mathbf{z}_4]^T$ is also Gaussian with probability density function (pdf)

$$f_{\mathbf{z}|v_{ij}}(z) \sim \mathcal{N}(\mu_i, C_{ij}), \quad (5)$$
$$\mu_i = [h_1\ h_2\ h_3\ h_4]^*E\{\mathbf{n}\} + e_i, \quad (6)$$
$$C_{ij}(k,l) = C(k,l) = h_k^*R_{\mathbf{nn}}h_l. \quad (7)$$

Here, e_i is the ith unit vector, and $R_{\mathbf{nn}}$ is the covariance matrix of $\mathbf{n}$. Note in this case C is independent of ij and $Pc_{ij} = Pc_{ik}\ \forall j,k$.

Case II: Non-Additive or Non-Gaussian Noise
In the general case, the exact pdf of $\mathbf{z}$ is difficult to find. However, if the noise spatial correlation decays rapidly enough, we can show that the correlator outputs are approximately Gaussian. If the noise is additive but non-Gaussian, the correlator output pdfs are approximately given by (5)–(7). For non-additive noise, the mean and covariance of this approximate Gaussian pdf are given by [5]:

$$\mu_{ij} = [h_1\ h_2\ h_3\ h_4]^*E\{\mathbf{y}|v_{ij}\} \quad (8)$$

$$C_{ij}(k,l) = h_k^*R_{\mathbf{y}_{ij}\mathbf{y}_{ij}}h_l, \quad (9)$$

where $R_{\mathbf{y}_{ij}\mathbf{y}_{ij}}$ is the covariance matrix of $\mathbf{y}$ given v_{ij}.

4. MODIFIED CORRELATORS FOR IMAGES WITH NONZERO MEAN NOISE

In many applications the noise in the test images have nonzero mean. For example, synthetic aperture radar (SAR) images are complex-valued, but typically only the magnitude of the images is used for pattern classification because of large phase deviations between the training and test patterns [5]. It is often reasonable to assume an additive Gaussian noise model for the complex measurements; this results in non-Gaussian, multiplicative noise in the corresponding magnitude images. In this section we modify the standard correlator designs to handle such multiplicative noise.

Assume the complex, noisy test image is given by $\mathbf{s} = v_{ij} + \mathbf{n}$, where v_{ij} is one of the complex training images from the ith class, and $\mathbf{n}$ is complex-valued noise. We form $\mathbf{y} = |\mathbf{s}|$, where the magnitude is computed at each pixel. The noise to the correlator, $|v_{ij} + \mathbf{n}| - |v_{ij}|$, is multiplicative and is not WSS since it is a function of the training images. A direct consequence is that the means of the correlator outputs are not equal to those corresponding to the noiseless training images, and classification performance degrades. If the output mean at the origin of the kth correlator is larger than the ith correlator output given ω_i (which can happen in practice [5]), the misclassification rate increases significantly.

To avoid the above problem, we propose a modified correlator filter design in which we replace the third filter design constraint in Section 2 by:

3. $E\{\mathbf{y}^* h_i | v_{kj}\} = \delta_{i-k}$, for all k = 1,...,I, and j = 1,...,n_k.

This modification is in keeping with the statistical pattern recognition goal of separating the ensembles of the correlator outputs corresponding to the noisy images, and thereby improves robustness to non-additive and non-zero-mean noise.

The optimization criteria for the filter designs are also modified correspondingly. Specifically, we replace equations (1)-(2) by:

$$h_i = A^{-1}\hat{m}(\hat{m}^* A^{-1}\hat{m})^{-1} u_i \quad (10)$$

$$H_i = d \cdot A^{-1}\hat{M}(\hat{M}^* A^{-1}\hat{M})^{-1} u_i \quad (11)$$

where $\hat{m}$ is a matrix of ensemble means of the noisy training vectors and $\hat{M}$ is its corresponding Fourier transform.

Three particular modified filters, corresponding to Table 1, are shown in Table 2; others can be found in [5]. For SDF_1 in Table 2, $A = I$. For $MVSDF_1$, $A = \sum_{i,j} R_{\mathbf{y}_{ij}\mathbf{y}_{ij}}$; for white noise A is thus diagonal. For $MACE_1$, A is a diagonal matrix whose diagonal elements are the average of $E\{|\mathbf{Y}|^2 | v_{ij}\}$, where $\mathbf{Y}$ is the frequency representation of $\mathbf{y}$.

Table 2: The modified SDF-type correlators

Correlator	Constraint	Criterion to minimize:
SDF_1	$\hat{m}^* h = u$	correlator energy: $h^* h$
$MVSDF_1$	$\hat{m}^* h = u$	average output var. at the origin: $h^* \hat{R}_{\mathbf{yy}} h$
$MACE_1$	$\hat{M}^* H = d \cdot u$	avg. expected corr. energy: $H^* \hat{D} H$

For the special case that $\mathbf{n}$ is complex Gaussian, analytic expressions for $\hat{m}$ and $\hat{R}_{\mathbf{yy}}$ can be obtained. In this case $\mathbf{y}$ is Rician distributed [5], with pdf

$$f_{\mathbf{y}|v_{ij}}(y|v_{ij}) = \frac{y}{\sigma^2} e^{-\frac{y^2+\eta_{ij}^2}{2\sigma^2}} I_0\left(\frac{y\eta_{ij}}{\sigma^2}\right), \quad (12)$$

where $\eta_{ij} = |v_{ij}|$, and $I_0(t) = \frac{1}{2\pi}\int_0^{2\pi} e^{t\cos\theta} d\theta$ is the *zero-order modified Bessel function of the first kind.* Let $\hat{m}_{ij} = E\{\mathbf{y}|v_{ij}\}$; then we have

$$\hat{m}_{ij} = \int_0^\infty \frac{y^2}{\sigma^2} e^{-\frac{y^2+\eta_{ij}^2}{2\sigma^2}} I_0\left(\frac{y\eta_{ij}}{\sigma^2}\right) dy. \quad (13)$$

For complex white noise, it is easy to show that

$$Var\{\mathbf{y}|v_{ij}\} + \hat{m}_{ij}^2 = 2\sigma^2 + \eta_{ij}^2, \quad (14)$$

where $2\sigma^2$ is the noise variance. In addition, for $MACE_1$ it can be shown [5] that

$$E\{|\mathbf{Y}|^2 | v_{ij}\} = |\hat{M}_{ij}|^2 + \sum_k Var\{\mathbf{y}(k)|v_{ij}\},$$

where $\mathbf{y}(k)$ is the kth element of $\mathbf{y}$.

For colored Gaussian noise, the means of the elements of $\mathbf{y}$ remain the same (see equation (13)), and the auto-covariances and cross-covariances of elements of $\mathbf{y}$ can be obtained by second and forth order numerical integrations of the joint density of the associated elements.

For non-Gaussian noise, or as an alternative to analytical expressions for Gaussian noise, one can estimate $\hat{m}_{ij}$ and $Var\{\mathbf{y}|v_{ij}\}$ using standard sample estimates. Specifically, for a given clutter model we generate N independent samples of the clutter to form noisy training patterns $\{y_k\}_{k=1}^N$ and compute the sample mean and sample variance in the usual way. This approach is often computationally attractive because it avoids numerical integrations; on the other hand, the resulting correlator performance is somewhat degraded, because this approach can be be regarded as applying a clutter model which is not exactly equal to the true clutter model to synthesize the correlators. Because the sample mean and covariance are consistent estimates, the degradation can be made arbitrarily small by choosing the sample size N large enough.

5. SIMULATION RESULTS

In these simulations, 11 training images from each of four classes are used; each image is a synthetically-generated (by X-Patch) SAR image of one of four vehicles each at one of eleven azimuth angles. The images are 64 × 64 pixels with 1 foot × 1 foot resolution. A leave-one-out criterion is employed in the simulations, *i.e.*, we synthesize the correlators by leaving one of the training patterns out, and test the correlator with the noisy training pattern which is left out. We use complex Gaussian noise, and we assume equal prior probabilities P_{ij}. The exact noise model is used to synthesize the correlators, so the performance obtained here is the best that the modified correlators can achieve. Figure 1 shows the correct detection rates of the standard and our modified correlators for complex white Gaussian noise. Both theoretical results and Monte-Carlo simluation results are shown. The theoretical curves are obtained by numerically integrating

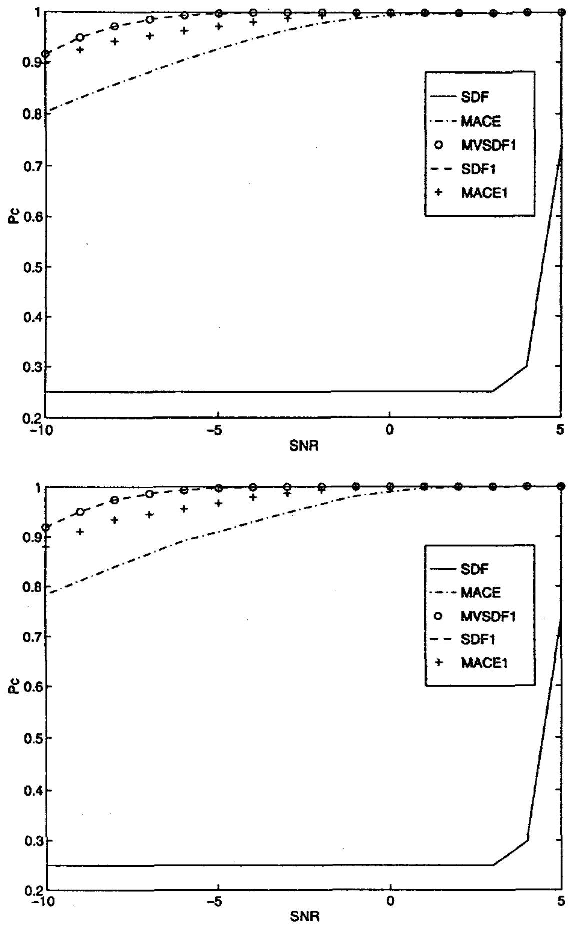

Figure 1: Correlator classification performance on magnitude images, using white Gaussian noise. Top: theoretical performance. Bottom: Monte-Carlo simulations

the theoretical detection rates in equation (5) using (8) and (9); they are approximate since the density of the correlator outputs at the origin are only approximately Gaussian. We see very good agreement between theoretical and simulation results, and we see greatly improved performance for SDF and MVSDF when the modification is employed. More detailed comparisons and interpretation can be found in [5]. Figure 2 shows the Monte Carlo simulation performance of the standard and our modified correlators for colored Gaussian noise with SNR=-10 dB, and 2D correlation function $R_{\mathbf{nn}}(k,l) = \rho^{\sqrt{k^2+l^2}}$, where $0 \leq \rho \leq 1$. For ρ=0 the noise is white, and for ρ close to 1 the noise is highly correlated. Sample means, covariances, and correlation energy are employed to synthesize the correlators. The results also show greatly improved performance for SDF when the modification is employed. The per-

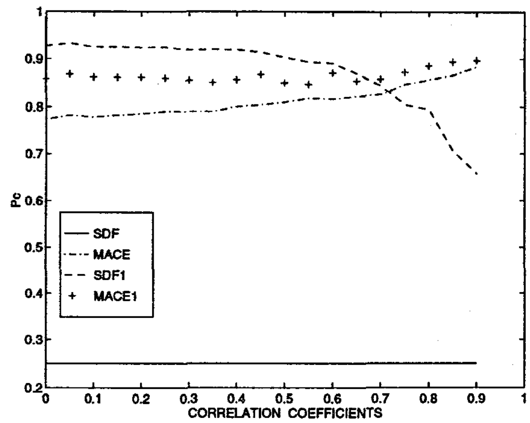

Figure 2: Monte Carlo simulation performance on magnitude images, using colored Gaussian noise

formance of $MVSDF_1$ is not computed because of the difficulty in realizing the 4096 × 4096 covariance matrices $R_{\mathbf{y}_{ij}\mathbf{y}_{ij}}$; its performance should be bounded below by the performance of SDF_1 and $MACE_1$.

6. CONCLUSIONS

We presented a theoretical analysis for the detection rates of linear correlator filters used for pattern recognition in the presence of additive and non-additive noise. We also developed a modified filter design method which improves the robustness of these linear correlators when applied to non-additive noise scenarios. This analysis has application in linear pattern recognition systems for both optical and synthetic aperture radar imagery.

7. REFERENCES

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