TABLE 9.1: Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$	
1	$\delta[k-j]$	$f[k]$	$f[k-j]$	
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma}\right] u[k]$	
3	$u[k]$	$u[k]$	$(k+1)u[k]$	
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right] u[k] \quad \gamma_1 \neq \gamma_2$	
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$	$\lvert\gamma_2\rvert > \lvert\gamma_1\rvert$
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1-\gamma_2}{\gamma_2} k\gamma_1^k\right] u[k]$	$\gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6}k(k-1)(k+1)u[k]$	
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$	
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right] u[k]$	
10	$\lvert\gamma_1\rvert^k \cos(\beta k+\theta)u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R}\left[\lvert\gamma_1\rvert^{k+1}\cos[\beta(k+1)+\theta-\phi] - \gamma_2^{k+1}\cos(\theta-\phi)\right] u[k]$ $R = \left[\lvert\gamma_1\rvert^2+\gamma_2^2-2\lvert\gamma_1\rvert\gamma_2\cos\beta\right]^{1/2}$ $\phi = \tan^{-1}\left[\frac{(\lvert\gamma_1\rvert\sin\beta)}{(\lvert\gamma_1\rvert\cos\beta-\gamma_2)}\right]$	γ_2 real

Note that

$$(4)^{-k}u[k] = \left(\tfrac{1}{4}\right)^k u[k] = (0.25)^k u[k]$$

Therefore

$$y[k] = (0.25)^k u[k] * (-0.2)^k u[k] + 4(0.25)^k u[k] * (0.8)^k u[k]$$

We use Pair 4 (Table 9.1) to find the above convolution sums.

$$\begin{aligned} y[k] &= \left[\frac{(0.25)^{k+1}-(-0.2)^{k+1}}{0.25-(-0.2)} + 4\frac{(0.25)^{k+1}-(0.8)^{k+1}}{0.25-0.8}\right]u[k] \\ &= \left(2.22\left[(0.25)^{k+1}-(-0.2)^{k+1}\right] - 7.27\left[(0.25)^{k+1}-(0.8)^{k+1}\right]\right)u[k] \\ &= \left[-5.05(0.25)^{k+1} - 2.22(-0.2)^{k+1} + 7.27(0.8)^{k+1}\right]u[k] \end{aligned}$$

Recognizing that

$$\gamma^{k+1} = \gamma(\gamma)^k$$

We can express $y[k]$ as

$$\begin{aligned} y[k] &= \left[-1.26(0.25)^k + 0.444(-0.2)^k + 5.81(0.8)^k\right]u[k] \\ &= \left[-1.26(4)^{-k} + 0.444(-0.2)^k + 5.81(0.8)^k\right]u[k] \end{aligned} \qquad (9.55)$$

■

△ **Exercise E9.7**

Show that $(0.8)^{k+1}u[k] * u[k] = 4[1-0.8(0.8)^k]u[k]$

Use convolution table. Recognize that $(0.8)^{k+1} = 0.8(0.8)^k$ ▽

△ **Exercise E9.8**

Show that $k\,3^{-k}u[k] * (0.2)^k u[k] = \frac{15}{4}[(0.2)^k - (1-\frac{2}{3}k)3^{-k}]u[k]$

Hint: Use convolution table. Recognize that $3^{-k} = (\frac{1}{3})^k$ ▽

△ **Exercise E9.9**

Using convolution table, show that $e^{-k}u[k] * 2^{-k}u[k] = \frac{2}{2-e}[e^{-k} - \frac{e}{2}2^{-k}]u[k]$

Hint: $e^{-k} = (\frac{1}{e})^k$ and $2^{-k} = (0.5)^k$ ▽

⊙ **Computer Example C9.4**

Find and sketch the zero-state response for the system described by

$$(E^2+6E+9)y[k] = (2E^2+6E)f[k]$$

for the input $f[k] = 4^k u[k]$.

```
k=0:11;
b=[2 6 0];
a=[1 6 9];
f=4.^(k);
y=filter(b,a,f);
stem(k,y)
xlabel('k');ylabel('y[k]');
```

⊙