Matemática Discreta

Mini-prova 1 - 2010.1

Prof. Juliano Iyoda

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1. $\{0.4pt\}$ Prove **por tabela verdade** que $\neg((q \wedge (q \vee p)) \vee \neg q) \equiv \mathsf{F}$.
   Resposta:

p	q	$q \vee p$	$q \wedge (q \vee p)$	$\neg q$	$(q \wedge (q \vee p)) \vee \neg q$	$\neg((q \wedge (q \vee p)) \vee \neg q)$
F	F	F	F	T	T	F
F	T	T	T	F	T	F
T	F	T	F	T	T	F
T	T	T	T	F	T	F

2. $\{0.8pt\}$ Prove **com equivalências lógicas** que

$$\neg(p \vee (p \wedge r)) \vee (\neg(q \vee \neg r)) \equiv (p \rightarrow \neg q) \wedge (p \rightarrow r).$$

   Justifique cada passo de prova com as equações 1 a 34 em anexo.
   Resposta:

$$\begin{aligned}
\neg(p \vee (p \wedge r)) \vee (\neg(q \vee \neg r)) &\equiv (\neg p) \vee (\neg(q \vee \neg r)) & [18]\\
&\equiv (\neg p) \vee (\neg q \wedge \neg\neg r) & [17]\\
&\equiv (\neg p) \vee (\neg q \wedge r) & [9]\\
&\equiv p \rightarrow (\neg q \wedge r) & [22]\\
&\equiv (p \rightarrow \neg q) \wedge (p \rightarrow r) & [27]
\end{aligned}$$

3. $\{0.8pt\}$ Prove **com equivalências lógicas** que

$$\forall x \exists y (P(x,y) \rightarrow (Q(x,y) \rightarrow \neg R(x,y))) \equiv \neg \exists x \forall y (P(x,y) \wedge (Q(x,y) \wedge R(x,y)))$$

   Justifique cada passo de prova com as equações 1 a **36** em anexo.
   Resposta:

$$\begin{aligned}
\forall x \exists y (P(x,y) \rightarrow (Q(x,y) \rightarrow \neg R(x,y))) &\equiv \forall x \exists y (P(x,y) \rightarrow (\neg Q(x,y) \vee \neg R(x,y))) & [22]\\
&\equiv \forall x \exists y (P(x,y) \rightarrow \neg(Q(x,y) \wedge R(x,y))) & [16]\\
&\equiv \forall x \exists y (\neg P(x,y) \vee \neg(Q(x,y) \wedge R(x,y))) & [22]\\
&\equiv \forall x \exists y (\neg(P(x,y) \wedge (Q(x,y) \wedge R(x,y)))) & [16]\\
&\equiv \forall x \neg \forall y (P(x,y) \wedge (Q(x,y) \wedge R(x,y))) & [36]\\
&\equiv \neg \exists x \forall y (P(x,y) \wedge (Q(x,y) \wedge R(x,y))) & [35]
\end{aligned}$$

$$\mathsf{T} \equiv \neg\mathsf{F} \tag{1}$$
$$\neg\mathsf{T} \equiv \mathsf{F} \tag{2}$$
$$p \wedge \mathsf{T} \equiv p \tag{3}$$
$$p \vee \mathsf{F} \equiv p \tag{4}$$
$$p \vee \mathsf{T} \equiv \mathsf{T} \tag{5}$$
$$p \wedge \mathsf{F} \equiv \mathsf{F} \tag{6}$$
$$p \vee p \equiv p \tag{7}$$
$$p \wedge p \equiv p \tag{8}$$
$$\neg(\neg p) \equiv p \tag{9}$$
$$p \vee q \equiv q \vee p \tag{10}$$
$$p \wedge q \equiv q \wedge p \tag{11}$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r) \tag{12}$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \tag{13}$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \tag{14}$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \tag{15}$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q \tag{16}$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q \tag{17}$$
$$p \vee (p \wedge q) \equiv p \tag{18}$$
$$p \wedge (p \vee q) \equiv p \tag{19}$$
$$p \vee \neg p \equiv \mathsf{T} \tag{20}$$
$$p \wedge \neg p \equiv \mathsf{F} \tag{21}$$
$$p \rightarrow q \equiv \neg p \vee q \tag{22}$$
$$p \rightarrow q \equiv \neg q \rightarrow \neg p \tag{23}$$
$$p \vee q \equiv \neg p \rightarrow q \tag{24}$$
$$p \wedge q \equiv \neg(p \rightarrow \neg q) \tag{25}$$
$$\neg(p \rightarrow q) \equiv p \wedge \neg q \tag{26}$$
$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r) \tag{27}$$
$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r \tag{28}$$
$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r) \tag{29}$$
$$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r \tag{30}$$
$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \tag{31}$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q \tag{32}$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q) \tag{33}$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q \tag{34}$$
$$\neg\exists x P(x) \equiv \forall x \neg P(x) \tag{35}$$
$$\neg\forall x P(x) \equiv \exists x \neg P(x) \tag{36}$$
$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array} \tag{37}$$
$$\begin{array}{l} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array} \tag{38}$$
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array} \tag{39}$$
$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array} \tag{40}$$
$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array} \tag{41}$$
$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array} \tag{42}$$
$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array} \tag{43}$$
$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array} \tag{44}$$
$$\begin{array}{l} \forall x P(x) \\ \hline \therefore P(c) \end{array} \tag{45}$$
$$\begin{array}{l} P(c)\textit{,para um c arbitrário} \\ \hline \therefore \forall x P(x) \end{array} \tag{46}$$
$$\begin{array}{l} \exists x P(x) \\ \hline \therefore P(c)\textit{,para algum c} \end{array} \tag{47}$$
$$\begin{array}{l} P(c)\textit{,para algum c} \\ \hline \therefore \exists x P(x) \end{array} \tag{48}$$
$$a \notin A \equiv \neg(a \in A) \tag{49}$$
$$\{x \mid x \in A\} = A \tag{50}$$
$$P(a) \equiv a \in \{x \mid x \in A\} \tag{51}$$
$$(A = B) \equiv \forall x(x \in A \leftrightarrow x \in B) \tag{52}$$
$$(A \subseteq B) \equiv \forall x(x \in A \rightarrow x \in B) \tag{53}$$
$$\begin{aligned} (A \subset B) \;\equiv\; & \forall x(x \in A \rightarrow x \in B) \wedge \\ & \exists x(x \in B \wedge x \notin A) \end{aligned} \tag{54}$$
$$\emptyset \subseteq S, \textit{para todo } S \tag{55}$$
$$S \subseteq S, \textit{para todo } S \tag{56}$$
$$(A \times \emptyset) = (\emptyset \times A) = \emptyset \tag{57}$$
$$A \cup B = \{x \mid x \in A \;\vee\; x \in B\} \tag{58}$$
$$(x \in A \;\vee\; x \in B) \;\equiv\; (x \in (A \cup B)) \tag{59}$$
$$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\} \tag{60}$$
$$(x \in A \;\wedge\; x \in B) \;\equiv\; (x \in (A \cap B)) \tag{61}$$
$$|A \cup B| = |A| + |B| - |A \cap B| \tag{62}$$
$$A - B = \{x \mid x \in A \;\wedge\; x \notin B\} \tag{63}$$
$$(x \in A \;\wedge\; x \notin B) \;\equiv\; (x \in (A - B)) \tag{64}$$
$$\overline{A} = \{x \mid x \notin A\} \tag{65}$$
$$(x \notin A) \;\equiv\; (x \in \overline{A}) \tag{66}$$
$$A \cup \emptyset = A \tag{67}$$
$$A \cap U = A \tag{68}$$
$$A \cup U = U \tag{69}$$
$$A \cap \emptyset = \emptyset \tag{70}$$
$$A \cup A = A \tag{71}$$
$$A \cap A = A \tag{72}$$
$$\overline{(\overline{A})} = A \tag{73}$$
$$A \cup B = B \cup A \tag{74}$$
$$A \cap B = B \cap A \tag{75}$$
$$A \cup (B \cup C) = (A \cup B) \cup C \tag{76}$$
$$A \cap (B \cap C) = (A \cap B) \cap C \tag{77}$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \tag{78}$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \tag{79}$$
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \tag{80}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B} \tag{81}$$
$$A \cup (A \cap B) = A \tag{82}$$
$$A \cap (A \cup B) = A \tag{83}$$
$$A \cup \overline{A} = U \tag{84}$$
$$A \cap \overline{A} = \emptyset \tag{85}$$