Matemática Discreta

Prova 1 - 2010.1

Prof. Juliano Iyoda

29 de Abril de 2010

Justifique cada passo de prova com exatamente 1 equação da lista em anexo. A única **exceção** a esta regra é o uso de comutatividade e associatividade. Ou seja, as equações (10), (11), (12), (13), (73), (74), (75) e (76) podem ser usadas simultaneamente em 1 passo de prova.

1. Sejam as equações

$$P(x) = x \notin x \qquad (85)$$
$$S = \{x \mid P(x)\} \qquad (86)$$

Prove o Paradoxo de Russell utilizando as equações de 1 a 86:

a) $\{1,5\ pt\}$ Dada a premissa $S \in S$, concluímos F.
Resposta:

1. $S \in S$	[Premissa]
2. $S \in \{x \mid P(x)\}$	[86]
3. $P(S)$	[48]
4. $S \notin S$	[85]
5. $\neg(S \in S)$	[46]
6. $(S \in S) \wedge \neg(S \in S)$	[43 em (1) e (5)]
7. F	[21]

b) $\{1,5\ pt\}$ Dada a premissa $S \notin S$, concluímos F.
Resposta:

1. $S \notin S$	[Premissa]
2. $\neg(S \in S)$	[46]
3. $\neg(S \in \{x \mid P(x)\})$	[86]
4. $\neg(P(S))$	[48]
5. $\neg(S \notin S)$	[85]
6. $\neg(\neg(S \in S))$	[46]
7. $S \in S$	[9]
8. $(S \in S) \wedge \neg(S \in S)$	[43 em (2) e (3)]
9. F	[21]

2. $\{1,0\ pt\}$ Sejam $g : A \to B$ e $f : B \to C$. Faça um desenho similar ao da Figura 1 retratando f e g de tal maneira que f **não** seja injetiva, mas tanto g quanto $(f \circ g)$ sejam injetivas.
Resposta:

3. Seja h uma função com domínio em R. Defina o conjunto imagem para:

a) $\{1,0\ pt\}$ $h(x) = \lfloor \lceil x \rceil \rfloor$. **Resposta:** Z

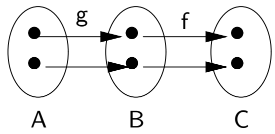

Figura 1: Exemplo de resposta.

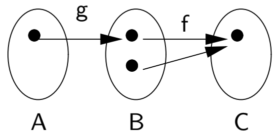

Figura 2: Resposta.

b) $\{1,0\ pt\}$ $h(x) = \lfloor x \rfloor + \lceil -x \rceil$. **Resposta:** $\{0\}$

4. $\{1,0\ pt\ EXTRA\}$ Prove que $((A - C) \cap (C - B)) = \emptyset$
Resposta:

$$
\begin{array}{rcll}
(A - C) \cap (C - B) & = & \{x \mid x \in (A - C) \wedge x \in (C - B)\} & [59] \\
& = & \{x \mid (x \in A \wedge x \notin C) \wedge (x \in (C - B)\} & [63] \\
& = & \{x \mid (x \in A \wedge x \notin C) \wedge (x \in C \wedge x \notin B)\} & [63] \\
& = & \{x \mid (x \in A \wedge x \notin B) \wedge (x \in C \wedge x \notin C)\} & [11,13] \\
& = & \{x \mid (x \in A \wedge x \notin B) \wedge (x \in C \wedge \neg(x \in C))\} & [46] \\
& = & \{x \mid (x \in A \wedge x \notin B) \wedge \mathsf{F}\} & [21] \\
& = & \{x \mid F\} & [6] \\
& = & \emptyset & [53]
\end{array}
$$

$$\mathsf{T} \equiv \neg \mathsf{F} \tag{1}$$
$$\neg \mathsf{T} \equiv \mathsf{F} \tag{2}$$
$$p \wedge \mathsf{T} \equiv p \tag{3}$$
$$p \vee \mathsf{F} \equiv p \tag{4}$$
$$p \vee \mathsf{T} \equiv \mathsf{T} \tag{5}$$
$$p \wedge \mathsf{F} \equiv \mathsf{F} \tag{6}$$
$$p \vee p \equiv p \tag{7}$$
$$p \wedge p \equiv p \tag{8}$$
$$\neg(\neg p) \equiv p \tag{9}$$
$$p \vee q \equiv q \vee p \tag{10}$$
$$p \wedge q \equiv q \wedge p \tag{11}$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r) \tag{12}$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \tag{13}$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \tag{14}$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \tag{15}$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q \tag{16}$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q \tag{17}$$
$$p \vee (p \wedge q) \equiv p \tag{18}$$
$$p \wedge (p \vee q) \equiv p \tag{19}$$
$$p \vee \neg p \equiv \mathsf{T} \tag{20}$$
$$p \wedge \neg p \equiv \mathsf{F} \tag{21}$$
$$p \rightarrow q \equiv \neg p \vee q \tag{22}$$
$$p \rightarrow q \equiv \neg q \rightarrow \neg p \tag{23}$$
$$p \vee q \equiv \neg p \rightarrow q \tag{24}$$
$$p \wedge q \equiv \neg(p \rightarrow \neg q) \tag{25}$$
$$\neg(p \rightarrow q) \equiv p \wedge \neg q \tag{26}$$
$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r) \tag{27}$$
$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r \tag{28}$$
$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r) \tag{29}$$
$$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r \tag{30}$$
$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \tag{31}$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q \tag{32}$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q) \tag{33}$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q \tag{34}$$
$$\neg \exists x P(x) \equiv \forall x \neg P(x) \tag{35}$$
$$\neg \forall x P(x) \equiv \exists x \neg P(x) \tag{36}$$
$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array} \tag{37}$$
$$\begin{array}{l} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array} \tag{38}$$
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array} \tag{39}$$
$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array} \qquad \begin{array}{l} p \vee q \\ \neg q \\ \hline \therefore p \end{array} \tag{40}$$
$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array} \qquad \begin{array}{c} p \\ \hline \therefore q \vee p \end{array} \tag{41}$$
$$\begin{array}{c} p \wedge q \\ \hline \therefore p \end{array} \qquad \begin{array}{c} p \wedge q \\ \hline \therefore q \end{array} \tag{42}$$
$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array} \tag{43}$$
$$\begin{array}{c} p \rightarrow q \\ \hline \therefore \neg q \rightarrow \neg p \end{array} \tag{44}$$
$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array} \tag{45}$$
$$a \notin A \equiv \neg(a \in A) \tag{46}$$
$$\{x \mid x \in A\} = A \tag{47}$$
$$P(a) \equiv a \in \{x \mid P(x)\} \tag{48}$$
$$(A = B) \equiv \forall x(x \in A \leftrightarrow x \in B) \tag{49}$$
$$(A \subseteq B) \equiv \forall x(x \in A \rightarrow x \in B) \tag{50}$$
$$\begin{aligned} (A \subset B) \quad \equiv \quad & \forall x(x \in A \rightarrow x \in B) \wedge \\ & \exists x(x \in B \wedge x \notin A) \end{aligned} \tag{51}$$
$$\emptyset \subseteq S, para\ todo\ S \tag{52}$$
$$\emptyset = \{x \mid \mathsf{F}\} \tag{53}$$
$$x \in \emptyset \equiv \mathsf{F} \tag{54}$$
$$S \subseteq S, para\ todo\ S \tag{55}$$
$$(A \times \emptyset) = (\emptyset \times A) = \emptyset \tag{56}$$
$$A \cup B = \{x \mid x \in A \ \vee \ x \in B\} \tag{57}$$
$$(x \in A \ \vee \ x \in B) \ \equiv \ (x \in (A \cup B)) \tag{58}$$
$$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\} \tag{59}$$
$$(x \in A \ \wedge \ x \in B) \ \equiv \ (x \in (A \cap B)) \tag{60}$$
$$|A \cup B| = |A| + |B| - |A \cap B| \tag{61}$$
$$A - B = \{x \mid x \in A \ \wedge \ x \notin B\} \tag{62}$$
$$(x \in A \ \wedge \ x \notin B) \ \equiv \ (x \in (A - B)) \tag{63}$$
$$\overline{A} = \{x \mid x \notin A\} \tag{64}$$
$$(x \notin A) \ \equiv \ (x \in \overline{A}) \tag{65}$$
$$A \cup \emptyset = A \tag{66}$$
$$A \cap U = A \tag{67}$$
$$A \cup U = U \tag{68}$$
$$A \cap \emptyset = \emptyset \tag{69}$$
$$A \cup A = A \tag{70}$$
$$A \cap A = A \tag{71}$$
$$\overline{(\overline{A})} = A \tag{72}$$
$$A \cup B = B \cup A \tag{73}$$
$$A \cap B = B \cap A \tag{74}$$
$$A \cup (B \cup C) = (A \cup B) \cup C \tag{75}$$
$$A \cap (B \cap C) = (A \cap B) \cap C \tag{76}$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \tag{77}$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \tag{78}$$
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \tag{79}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B} \tag{80}$$
$$A \cup (A \cap B) = A \tag{81}$$
$$A \cap (A \cup B) = A \tag{82}$$
$$A \cup \overline{A} = U \tag{83}$$
$$A \cap \overline{A} = \emptyset \tag{84}$$