Some Recent Theoretical Advances and Open Questions on Energy Consumption in Ad-Hoc Wireless Networks

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Abstract

One of the main benefits of power controlled *ad-hoc* wireless networks is their ability to vary the range in order to reduce the power consumption.
Minimizing energy consumption is crucial on such kind of networks since, typically, wireless devices are portable and benefit only of limited power resources.
On the other hand, the network must have a sufficient degree of connectivity in order to guarantee fast and efficient communication.

These two aspects yield a class of fundamental optimization problems, denoted as *range assignment* problems, that has been the subject of several works in the area of wireless network theory. The primary aim of this paper is thus to describe the most important recent advances on this class of problems. Rather than completeness, the paper will try to provide results and techniques that seem to be the most promising to address the several important related problems which are still open. Discussing such related open problems are indeed our other main goal.

1 Introduction and Motivations

During the last decade, *wireless networks* faced a tremendous development in the networking applications, mostly caused by the recent drop in equipment prices. Traditionally, wireless networks where implemented in LANs (Local Area Networks) [16], like an office environment where they bring flexibility for the users. The most recent WANs (Wide Area Networks) implementation opens new possibility in networking [5]. The major advantage of *ad-hoc wireless* networks relies on the *needless of an infrastructure*: the network is simply a collection of *hosts*, that is, radio transmitter/receivers which can communicate with each other by sending/receiving radio signals. In general, message communication takes place via *multi-hop* transmission, that is, a message is sent to its destination through a set of intermediate hosts. This is due to the fact that, because of radio signal propagation [21], an host may not be able to directly communicate with another one. Indeed, the *transmission power* of the sender, its *distance* to the receiver, and other *environmental conditions* determine whether the received radio signal is strong enough in order to make message decoding possible. The attenuation of a signal transmitted with power P_s equals to

$$P_r = \frac{P_s}{\texttt{dist}(s,t)^\alpha},$$

where $\texttt{dist}(s,t)$ denotes the distance between the two hosts s and t ([23]), and $\alpha \geq 1$ is the *distance-power gradient*. In addition, a message can be correctly decoded whenever $P_r \geq \gamma$, where the constant $\gamma \geq 1$ is the *transmission-quality* parameter. Therefore, the so called *transmission range* of a station is determined by its transmission power: the (maximum) distance a station s can transmit to is bounded by $(P_s/\gamma)^{1/\alpha}$, where P_s is the (maximum) power s can transmit with.

Notice that, in some cases hosts are portable devices which benefits only of limited power resources. One of the main benefits of ad-hoc *power controlled* networks is the ability of the hosts to vary the power used in the transmission (and therefore the transmission range) in order to avoid interference problems and reduce the power consumption.

As we will see in the sequel, deciding the transmission power of the single hosts in order to (i) guarantee a "good" communication between hosts, and (ii) minimize the overall power consumption of the network,

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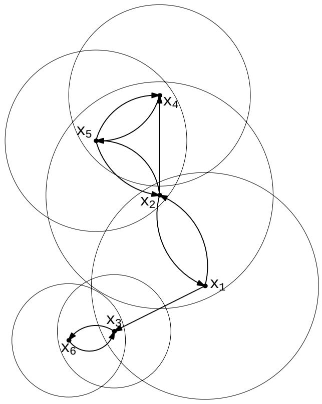

Figure 1: Example of a communication graph

gives rises to interesting algorithmic questions. In particular, these two aspects yield a class of fundamental optimization problems, denoted as *range assignment* problems, that has been the subject of several works in the area of wireless network theory. The primary aim of this work is thus to describe the most important recent advances on the problems mentioned above. Rather than completeness, this paper will try to provide results and techniques that seem to be the most promising to address the several important related problems which are still far to be solved. Discussing the related open problems is indeed the other main goal of this work.

Range Assignment Problems. In what follows, we provide a formal definition of the range assignment problems. Stations are here considered as points of the d-dimensional Euclidean space.

Given a set of stations S, a *range assignment* for S is a function $r : S \to \mathbb{R}^+$. The *cost* of a range assignment r is the overall power consumption, that is

$$\texttt{cost}(r) = \sum_{v \in S} [r(v)]^\alpha.$$

A range assignment r for a set S of stations yields a directed *communication graph* $G_r = (S, E)$ such that, for each $(u, v) \in S \times S$, the directed edge (u, v) belongs to E if and only if v is at distance at most $r(u)$ from u. Notice that here we have fixed $\gamma = 1$, however, all the results hold for any $\gamma \geq 1$.

An ad-hoc wireless network can then be viewed as a communication graph associated with a certain range assignment as shown in Figure 1.

Depending on the particular application, the communication graph is required to satisfy a given property Π. By varying property Π, we can obtain the class of range assignment problems.

Definition 1 *Given a graph property* Π*, the* MIN-RANGE(Π) *problem is defined as follows*

Input: *A set of points* S.

Output: *A range assignment* r *for* S *such that* G_r *satisfies* Π *and* $\texttt{cost}(r)$ *is minimized.*

The range assignment problems that have attracted the attention of researchers are mainly those in which the graph property enables to implement a network primitive. Such properties are listed below.

- *strong connectivity* (SC); G_r must be strongly connected.
- *h-strong connectivity* (hSC); From every stations s to any other t, G_r must contain a directed path of length at most h;
- *broadcast* (B), given a particular node s (called the *source*), G_r must contain a directed source spanning tree rooted at s;
- *h-broadcast* (hB), as in the previous case but with the further property that the source tree must have depth at most h.

In the next sections, we will see that the computational complexity of the above range assignment problems varies dramatically depending on the property Π and on the other parameters of the wireless networks, such us the distance-power gradient α and the dimension of the Euclidean space the network is located on. As mentioned before, this paper will review the main complexity results and open theoretical questions related to the above problems.

	$d = 1$	$d = 2$	$d > 2$
$\alpha = 1$	$\in$ P ([18])	$2-$ APX ([18])	NP-hard ([18]) $2-$ APX ([18])
$\alpha \geq 2$	$\in$ P ([18])	NP-hard ([14]) $2-$ APX ([18])	APX-hard ([14]) $2-$ APX ([18])

Table 1: Complexity of the MIN-RANGE(SC) problem based on the dimension d and the distance power gradient α.

2 The Strong Connectivity

MIN-RANGE(SC) was the first studied problem in this area [18]. The importance of this problem is due to the fact that, in a wireless network, it can be useful that every station can communicate with all the other ones (the complexity results regarding this problem are summarized in Table 1).

Strong Connectivity on the Line. When the stations are located along a line, the network is said *linear*. Linear radio networks have been the subject of several recent studies [4, 8, 17, 22]. As pointed out in [22], rather than a simplification, this version of the problem results in a more accurate analysis of the situation arising, for instance, in vehicular technology applications. It is common opinion to consider one-dimensional frameworks as in fact the most suitable ones in studying road traffic information systems [4, 17, 20, 22]. Vehicles follow roads, and messages are to be broadcasted along lanes. Typically, the curvature of roads is small in comparison to the transmission range (half a mile up to some few miles).

Kirousis *et al* [18] showed that the MIN-RANGE(SC) problem restricted to linear instances can be solved in polynomial time.

Theorem 2 ([18]) *There exists an algorithm that finds an optimal solution for* MIN-RANGE(SC) *in* $O(n^4)$ *time when its input is a set* S *of* n *points in a 1-dimensional Euclidean space,*

The algorithm makes a rather involved use of dynamic programming and it is thus omitted here (all the details of the proof can be found in [18]). However, as we will see in the next sections, the use of dynamic programming characterizes all the efficient algorithms for the range assignment problems when the input network is linear.

Strong Connectivity on Higher Dimensions. When stations are spread on a multidimensional space, finding an optimal solution for the MIN-RANGE(SC) problem is computationally hard.

Theorem 3 (Hardness results) *For any distance-power gradient* $\alpha > 1$*, the* MIN-RANGE(SC) *problem is* NP*-hard for* $d \geq 2$ [18, 13]*; it is* APX*-hard for* $d \geq 3$ [13]*, so it does not admit a PTAS unless* P = NP.

The above hardness results are all proved by using reductions from variants of the MIN VERTEX COVER problem. The interested reader may consult the complete proofs in [18] and [13].

Theorem 3 shows that the MIN-RANGE(SC) problem is hard to solve exactly and, for $d \geq 3$, it does not admit a PTAS unless P = NP. On the other hand, Kirousis *et al* [18] derived a simple and efficient 2-approximation algorithm for MIN-RANGE(SC), that is, an algorithm that, given any instance of the problem, returns a solution whose cost is at most twice the optimum. The algorithm MSTALG is based on the computation of a minimum spanning tree `mst`.

begin
T := $\texttt{mst}(S, \texttt{dist})$;
forall $v \in S$ **do**
$r^{\texttt{mst}}(v)$:= $\max_{u:(v,u)\in T}\{\texttt{dist}(v, u)\}$;
end

Figure 2: The MSTALGalgorithm.

Given a set $S = \{x_1, \ldots, x_n\}$ of points in the d-dimensional Euclidean space, consider the complete weighted graph $G = (S, E)$ where the weight of the edge $\{x_i, x_j\}$ is set to $\texttt{dist}(x_i, x_j)$. The algorithm is described in Figure 2. Clearly this algorithm runs in $O(n^2)$ time and returns a feasible range assignment.

Theorem 4 ([18]) *For any instance* S*,* MSTALG *returns a range assignment* $r^{\texttt{mst}}(S)$ *such that*

$$\texttt{cost}(r^{\texttt{mst}}(S)) \leq 2 \cdot \texttt{opt}_{\texttt{sc}}(S)$$

where $\mathtt{opt}_{sc}(S)$ *is the optimal cost.*

The MstAlg algorithm also works when the input is a generic graph since the quality of the solution relies only on the properties of minimum spanning trees.

Strong Connectivity with Symmetry. In [7, 6] another version of the Min-Range(SC) problem is proposed. Here, in addition to the strong connectivity property, the communication graph must be symmetric. In the two papers it is observed that the proof of NP-hardness in [13] (Theorem 3) also holds for the symmetric case. Moreover, in [7] a 15/8 approximation algorithm is given.

Open Problems. It is unknown whether Min-Range(SC) in the 2-dimensional Euclidean space is APX-hard or admits a PTAS. A more sophisticated reduction than those shown in [13, 18] might yield again an APX-hardness result. Another challenging goal is to improve the approximation factor achieved by the MstAlg algorithm at least in some interesting restrictions of the problem. About this issue, we observe that the MstAlg algorithm does not exploit at all the Euclidean properties of the instance, so we believe that a better algorithm should rely on such Euclidean properties. A seemingly simpler case is that in which $\alpha = 1$. Since the communication graph satisfies the triangular inequality, we conjecture that this case is efficiently solvable or, at least, approximable within a factor smaller than 2. As for linear network, a relevant future work is that of designing a more efficient polynomial-time algorithm.

3 The h-Strong Connectivity

In the Min-Range(hSC) problem, the feasible solutions must yield a communication graph having (directed) diameter bounded by h.

The complexity of the Min-Range(hSC) problem is still unknown. Clearly, the problem is NP-hard for multi-dimensional instances since Min-Range(SC) is the special case of Min-Range(hSC) in which $h = n - 1$. However, nothing is known for other values of the parameter h (say, asymptotically smaller than n).

In what follows, we review and discuss some recent results concerning special cases of this problem.

h-Strong Connectivity on the Line. In [18], the following bounds on the optimal cost have been proved for a strong restriction of Min-Range(hSC) on the line.

Theorem 5 (The Uniform Chain Case [18]) *Let S be a set of n points equally spaced at distance $\delta > 0$ on the same line; let $\mathtt{opt}_{hsc}(S)$ be the cost of an optimal solution for* Min-Range(hSC) *on input h and S. Then, it holds that*

1. $\mathtt{opt}_{hsc}(S) = \Theta\left(\delta^2 n^{\frac{2^{h+1}-1}{2^h-1}}\right)$, *for any fixed positive integer h;*

2. $\mathtt{opt}_{hsc}(S) = \Theta\left(\delta^2 \frac{n^2}{h}\right)$, *for any $h = \Omega(\log n)$.*

Furthermore, the two above (implicit) upper bounds can be efficiently constructed.

The approximation ratio guaranteed by the first result of Theorem 5 *increases with h*. In [8], the following efficient algorithm is given

Theorem 6 ([8]) *There exists an algorithm that, given any linear wireless network and any $h > 0$, guarantees a 2-approximation ratio and runs in $O(hn^3)$ time.*

The same paper also provides a better approximation algorithm that works on any family of *well spaced* instances and for any constant h; in such instances, the ratio between the maximum and the minimum distance among adjacent stations is bounded by a polylogarithmic function of n. More precisely, it is shown that, for any well spaced instance and for any constant h, it is possible to compute in $O(hn^3)$ time a solution whose cost is at most $(1 + \epsilon(n))$ times the optimum, where $\epsilon(n) = o(1)$.

The above approximability results are obtained by exploiting exact solutions for two natural variants of the Min-Range(hSC) problem that may be of independent interest:

Min All-To-One Assignment Given a set S of stations on the line, a *sink* station $t \in S$, and an integer $h > 0$; find a minimum cost range assignment for S ensuring that any station is able to reach t in at most h hops.

Min Assignment with Bases Given a set S of stations on the line, and an integer $h > 0$; find a minimum cost range assignment for S such that, any station in S is either a *base* (a station is a base if it directly reaches any other station in S) or it reaches a base in at most $h - 1$ hops.

For each of the two above problems, they provide a dynamic programming algorithm that returns an optimal solution in $O(hn^3)$ time.

Finally, it is also proved that, for $h = 2$, the Min-Range($h\texttt{SC}$) problem can be solved in $O(n^3)$ time. This result is obtained by combining the algorithm for the Min Assignment with Bases problem with a simple characterization of the structure of any optimal 2-hops range assignment.

h-Strong Connectivity on Higher Dimensions. In [15, 14], the authors provide a lower and an upper bound on the optimal cost of any 2-dimensional instance, when h is an arbitrary constant. The results are given for $\alpha = 2$ but they hold for any $\alpha \geq 1$.

Given a set of stations S, let us define

$$D(S) = \max\{\texttt{dist}(s, s') \mid s, s' \in S\}; \quad \delta(S) = \min\{\texttt{dist}(s, s') \mid s, s' \in S,\ s \neq s'\}$$

and let $\texttt{opt}_h(S)$ be the cost of an optimal solution for the set S and for h hops.

Theorem 7 ([15, 14]) *For any constant $h > 0$, and for any set S of stations on the plane, it holds that*

$$\texttt{opt}_h(S) = \Omega(\delta(S)^2 |S|^{1+1/h}),$$

The same papers provide an efficient solution for any instance, for constant values of h:

Theorem 8 ([15, 14]) *For any set of stations S on the plane, it is possible to construct in time*[1] *$O(|S|)$ a feasible range assignment $r_h(S)$ such that*

$$\textsf{cost}(r_h(S)) = O(D(S)^2 |S|^{1/h}),$$

for any constant $h > 0$.

Let us now consider the planar configuration G_n where n stations are placed on a square grid of side $\sqrt{n}$ and the distance between adjacent pairs of stations is 1 (notice that this is the 2-dimensional version of the unit chain case studied in [18] - see Theorem 5). Then, by combining Theorem 7 and Theorem 8, we easily obtain the following optimal bound

$$\texttt{opt}_h(G_n) = \Theta\left(n^{1+1/h}\right). \tag{1}$$

The square grid configuration is the most regular case of *well-spread* instances. In general, we say that a family $\mathcal{S}$ of 2-dimensional instances is *well-spread* if, for any $S \in \mathcal{S}$, $\delta(S) \geq cD(S)/\sqrt{|S|}$, for some positive constant $c > 0$. Notice that the above property is rather natural: informally speaking, in a well-spread instance, any two stations must be not "too close". Because of interference problems, this is the typical situation in radio networks adopted in practice [22, 23]. It turns out that the optimal bound in Equation 1 holds for any family of well-spread instances. The following corollary is thus an easy consequence of Theorems 7 and 8.

Corollary 9 *Let $\mathcal{S}$ be a family of well-spread instances. For any $S \in \mathcal{S}$, it holds that*

$$\texttt{opt}_h(S) = \Theta\left(\delta(S)^2 |S|^{1+1/h}\right),$$

for any positive constant h.

Beside being interesting in itself, the well-spread concept turns out to be useful to analyze another important family of instances: the *random instances.* In fact, it is not hard to show that a family $\mathcal{S}^R$ of uniformly distributed random instances, with high probability, does not satisfy the well-spread property. However, in [14], it is shown that, given a family $\mathcal{S}^R$ of random instances, it is possible to construct a family $\mathcal{S}^W$ of well-spread instances having the following property: For any $S^r \in \mathcal{S}^R$, there is an $S^w \in \mathcal{S}^W$ such that $|S^w| = \Theta(|S^r|)$ and, with high probability, $\texttt{opt}_h(S^r) = \Theta(\texttt{opt}_h(S^w))$. This equivalence yields the following result.

[1] The constant hidden by the O notation is linear in h.

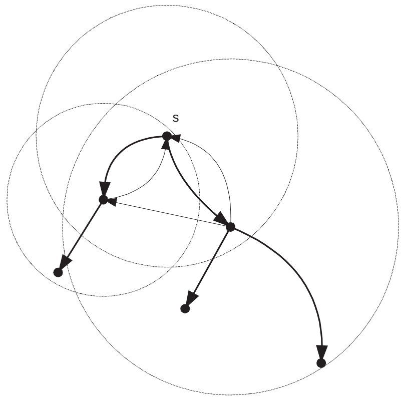

Figure 3: The spanning tree (bold arrows) from the source s of the transmission graph induced by the range assignment represented by the dashed circles.

Theorem 10 ([14]) *Let ℓ be any positive real. Let S^r be a set of n stations chosen uniformly and independently at random on a square of side ℓ. Then, with high probability, it holds that*

$$\mathtt{opt}_h(S^r) = \Theta\left(\ell^2 n^{1/h}\right),$$

for any constant h.

The lower bound obtained in Theorem 7 holds *for any instance*, so the constructive (and efficient) method of Theorem 8 and the equivalence yielding Theorem 10 easily imply the following result. Let Av-APX be the class of optimization problems (together with a probability function on the instance set) that admit a polynomial time algorithm that, with high probability, returns a feasible solution having performance ratio bounded by a fixed constant [1].

Corollary 11 ([14])

- *Let $\mathcal{S}$ be any family of well-spread instances. Then, the* MIN-RANGE(hSC) *problem restricted to $\mathcal{S}$ admits a polynomial-time approximation algorithm with constant performance ratio (i.e. the restriction is in* APX*), for any constant $h > 0$.*
- *The* MIN-RANGE(hSC) *problem (with uniform input probability) is in* Av-APX*, for any constant $h > 0$.*

Open Problems. Several questions related to MIN-RANGE(hSC) are still open. We here discuss only our favorite ones. As for the multi-dimensional case, we conjecture that the problem remains NP-hard even for constant values of h and for any $\alpha \geq 1$. We believe that a good starting point, in order to prove this conjecture, might be that of considering suitable versions of Facility Location Problems, optimization problems which are known to be NP-hard. As for the linear case, it is not known whether the problem is NP-hard for some range of h. We conjecture the existence of a dynamic-programming algorithm that solves the linear case in $O(n^{O(h)})$ time, for any $h > 0$. Notice that the problem is still unsolved for general instances even for $\alpha = 1$.

4 The Broadcast

Broadcast is a fundamental task and it represents a major part of the activities in real life multi-hop radio networks [2, 3]. In particular, it consists in a transmission of a message from a *source* station s to *all* stations in the wireless network.

The broadcast operation is achievable if the transmission graph induced by the range assignment contains a directed spanning tree rooted at the source station s (see Figure 3). It is thus well motivated the study of the MIN-RANGE(B) problem that, given a set of stations and a source station s, consists in finding a range assignment r such that its induced transmission graph contains a spanning tree rooted at s.

The MIN-RANGE(B) problem was introduced in [25] for the 2-dimensional case and when $\alpha = 2$. In this work, the performances of three heuristics have been experimentally compared (one to each other) on the random uniform model without providing theoretical results.

Some theoretical analysis of this problem has been presented in the papers [10, 9, 12, 24] where both positive and negative results are given. Table 4 summarizes these results. In particular MIN-RANGE(B) is computational intractable on multi-dimensional spaces.

	$d=1$	$d=2$	$d>2$
$\alpha=1$	$\in$ P (*folklore*)	$\in$ P (*folklore*)	$\in$ P (*folklore*)
$\alpha \geq 2$	$\in$ P ([12])	NP-hard ([10]) $3^{\alpha/2}2^{\alpha}$-APX([24])	NP-hard ([10])
$2<\alpha<d$	$\times$	$\times$	NP-hard ([10])
$\alpha \geq d$	$\times$	$\times$	NP-hard ([10]) $2^{O(d)}$-APX ([10])

Table 2: Complexity of the MIN-RANGE(B) problem based on the dimension d and the distance power gradient α.

Theorem 12 ([10]) *In the Euclidean d-dimensional space* MIN-RANGE(B) *is* NP*-hard for any $\alpha > 1$ and $d \geq 2$.*

On the other hand, when $\alpha = 1$ the problem is trivially in P because it suffices to set the range of the source s as the maximal distance between itself and any other stations. In [10], this result is just stated because its proof is very similar to the proof of Theorem 3. A complete proof is available in the technical report [9].

Broadcast on the Line. The MIN-RANGE(B) problem in the line is in P for any $\alpha > 0$. Indeed, a dynamic programming algorithm for the case $\alpha \geq 2$ is presented in [12] while the case $\alpha = 1$ is trivial. The case $\alpha \geq 2$ is not easily computable starting from smaller instances because the introduction of a new station could affect the previously computed ranges. An example can clarify the ideas: Consider the Figure 4, let $\alpha = 2$ and s be the source node, the optimal range assignment of Figure 4a has cost 3. If we add a station c on the right extreme of the line at distance 4 from the nearest station r (see Figure 4b), we have to assign a range 4 to r. But now, from r we can reach also the stations b and ℓ so the old ranges of a and b are no longer necessary. The cost of this optimal solution is 17.

If, in the optimal solution, there exists a situation similar to that shown in Figure 4b, i.e, a link exists from a station r in the right side of s towards the station ℓ in the left side of s (or vice-versa), we say that this optimal solution contains a *right-bridge* (or *left-bridge* if the symmetric situation holds). The algorithm in [12] is based on the following result.

Theorem 13 ([12]) *All the optimal solutions for the* MIN-RANGE(B) *problem in the line contains at most one right-bridge or one left-bridge.*

The algorithm also works for the MIN-RANGE(hB) problem. Its time complexity is $O(hn^2)$ where n is the number of stations and h is the maximum number of allowed hops.

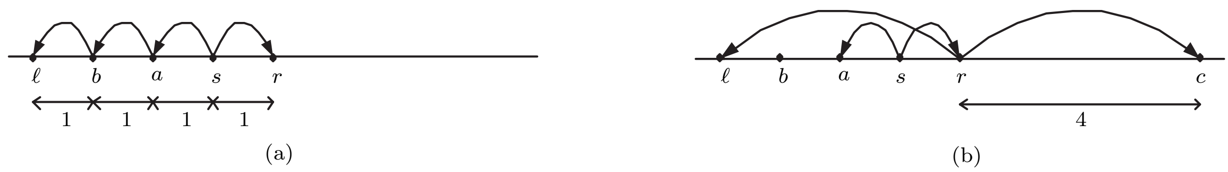

Figure 4: The greedy approach does not work. Indeed, the station c requires a right bridge that affects the assignment of the other stations.

Broadcast on the Plane. Theorem 12 states that MIN-RANGE(B) is NP-hard for any $d \geq 2$ and $\alpha > 1$. As previously mentioned, in [25] three heuristics are compared via simulation on random instances for the case $\alpha = 2$. The best heuristic appears to be the one based on the construction of an Euclidean *Minimum Spanning Tree* (`mst`) rooted at the source node. In particular, given a set of stations S in the Euclidean plane and a source node s, the MSTALG algorithm is defined as follows:

1. computes an undirected `mst` T according to the relative distance between pairs of stations;
2. roots T at s (this makes T directed);
3. for each station u sets $r^{\texttt{mst}}(u) = \max\{\texttt{dist}(u, v) \text{ for all } v : (u, v) \in T\}$.

Notice that MstAlg is the same algorithm that guarantees an approximation factor 2 for the Min-Range(SC) described in Section 2. The only difference consists in the orientation of the tree starting from the source s. The cost of the MstAlg solution with input S and s is defined as:

$$\texttt{cost}(\text{MstAlg}(\langle S, s\rangle)) = \sum_{v \in S} r^{\text{mst}}(s)^{\alpha}.$$

Theoretical upper bounds on the approximation performances of the MstAlg have been presented in [10, 24]. In the Euclidean plane, the analysis of the first paper proves an approximation factor of $10^{\alpha/2}2^{\alpha}$, whereas the approximation factor proved in the second paper is smaller: $3^{\alpha/2}2^{\alpha}$. However, the same authors of [10] improve their analysis in the technical report [9] leading the performance ratio to $5^{\alpha/2}2^{\alpha}$.

Informally speaking, all the above worst case approximation analysis for the MstAlg algorithm rely on the following interesting result.

Theorem 14 ([10, 9, 24]) *Let S be a set of points in the Euclidean space,* diam *be the diameter of the smaller disk containing S and $G^{\alpha} = (S, E)$ be the complete weighted undirected graph in which the weight of the edge (u, v) is defined as* $\texttt{dist}(u, v)^{\alpha}$*. Then, there exists a constant c depending on alpha such that*

$$\texttt{cost}(\texttt{mst}(G^{\alpha})) = c \cdot \mathsf{diam}^{\alpha}.$$

Three upper bounds for the constant c have been derived: $c \leq 10^{\alpha/2}$ [10]; $c \leq 5^{\alpha/2}$ [9] and $c \leq 3^{\alpha/2}$ [24]. The following theorem shows how the upper bounds for c can be used in order to obtain an approximation results.

Theorem 15 ([10, 9, 24]) *The* Min-Range(B) *problem in the Euclidean plane is approximable within a factor $c \cdot 2^{\alpha}$, for every $\alpha \geq 2$.*

Broadcast on Higher Dimensions. In [10] it is shown that the algorithm MstAlg achieves a constant (i.e. independent of the graph size) approximation ratio even on higher dimensions.

Theorem 16 ([10]) *There exists a function $f : \mathbb{N} \times \mathbb{R} \to \mathbb{R}$ such that, for any $d \geq 2$ and for any $\alpha \geq d$,* Min-Range(B) *in the d-dimensional Euclidean space is approximable within factor $f(d, \alpha)$.*

On the other hand, it can be proved that the function f in the above theorem grows exponentially with respect to d [10].

Open Problems. It is easy to construct an instance in the plane in which the MstAlg algorithm returns a solution whose cost is 6 times the cost of the optimal solution [10, 24]. On the other hand, from [24] it follows that, when $\alpha = 2$, MstAlg guarantees a 12 approximation on the plane. Then, a natural goal is to reduce the gap between these two bounds. We claim that the MstAlg is more efficient. Our claim is supported by the experimental results in [11]: On random instances the performance ratio achieved by MstAlg is at most 3.3. Finally, it is an open question whether Min-Range(B) admits a polynomial time approximation scheme.

5 The h-Broadcast

Another important range assignment problem is the one in which, a source station s is given, and the resulting communication graph must contain a source spanning tree of depth at most h. This problem is denoted as Min-Range(hB). Since Min-Range(B) can be seen as the restriction of Min-Range(hB) where $h = n - 1$, the latter is NP-hard in any multi-dimensional space (see Theorem 12). However, it is still unknown whether, for some (non trivial) ranges of h, the problem is efficiently solvable. The only positive result for this problem concerns the linear case. Indeed, as previously observed in Section 4, the dynamic algorithm proposed in [12] works for the h-broadcast as well.

It is also interesting to mention that [8] provides a dynamic programming algorithm that optimally solves the *all-to-one* version of the min-range assignment problem, i.e., the case in which Π consists in requiring that, for each station v, the range assignment must guarantee the existence of a path of length at most h from v to a *sink* s given in input. Their algorithm works in $O(hn^3)$ time. Informally speaking, this is the opposite version of the Min-Range(hB) problem. Even though, at a first glance, the problems seem to be mutually related, this is not the case: in particular, the latter problem cannot be reduced to the all-to-one version. Among the others, we emphasize two key-differences. *i*). In the all-to-one version, any feasible solution must assign a positive range to *every* station: this does not clearly hold for the broadcast version. *ii*). On the other hand, bridges are not useful for the all-to-one version while, as discussed above, it is a crucial issue in the Min-Range(hB) problem.

Open Problems. The real complexity of all versions of the Min-Range(hB) problem is still unknown but the linear case. Among the others, we find very interesting the following two cases:

- The 2-dimensional case in which $\alpha = 1$ and/or h is set to some small constant. We believe that, in this case, the use of a suitable combination of geometrical arguments and dynamic programming could yield efficient optimal algorithms.
- The multi-dimensional case in which $\alpha > 1$ and h is bounded by some small constant. The intuition here is that the problem remains hard to solve and only approximating solutions can be found in polynomial time.

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