A multiobjective approach of the virtual topology design and routing problem in WDM networks

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***Abstract*— We deal with the classical virtual topology design and routing problems in optical WDM (Wavelength Division Multiplexing) networks. We propose a multiobjective based algorithm to compute the Pareto set of solutions of the problem. Although the computational cost may be high, such approach permits the decision maker to have a better perception of the gain and the loss of choosing any given solution.**

We describe briefly the treated problem, and the MILP (Mixed Integer Linear Programing) model used. We present the method applied to obtain the Pareto set. We report some computational results and they fully justify the interest of carrying out a multiobjective study.

Keywords: Virtual topology design problem, routing problem, optical WDM network, multiobjective algorithm, Pareto set, MILP.

I. Introduction

Optical technology is a very flexible and powerful solution for transmitting information. It offers very large bandwidth, low energy consumption and dissipation. The use of *WDM* (*Wavelength Division Multiplexing*) technology allows the transmission of various signals in the same medium; each signal being modulated in an independent wavelength.

The measure of the efficiency in the use of a network is a key point. However, various *metrics* can be used; and improving performance with a given metric can lead to a decrease of performance with other metrics. Various criteria are used depending on the problem considered, and as far as we know there is no “universal” metric. Up until now the choice of a metric is made *a priori*, before the beginning of the optimization process. This method lacks of flexibility and lets the decision maker face a problem to be solved - the choice of a metric - *before* knowing the results of the optimization process.

Multiobjective optimization avoids this drawback: it does not compute an unique solution, but a set of solutions. Each one belongs to the *Pareto frontier* which represents the set of all “best” (non-dominated) points. Carrying out such an analysis can provide a significant amount of information - the relationship between metrics, *after* the optimization process. As far as we know there are few multiobjective works in telecommunication network field.

In the second section, we describe quickly the considered problem and the mathematical formulation we use. We then describe how we identify the Pareto frontier and which tests are carried out.

II. The VTDR problem

The Virtual Topology Design and Routing (VTDR) problem is one of the key problems in the design of a WDM network. It consists of defining the virtual topology, the wavelength allocation and the routing of the demands [1], [2], [3].

A. Description and hypothesis

A WDM network is composed of different layers: the *physical layer* and the *logical* (or *virtual*) *layer*. The latter is composed by *lightpaths*, which are direct or indirect connections between a pair of nodes. The virtual topology is the *communication graph* used to transport the traffic. Lightpaths, and consequently the virtual topology, are set through the configuration of the devices of the physical layer.

The Virtual Topology Design problem consists of defining a virtual topology by finding a set of lightpaths adapted to our needs. The routing problem consists of routing traffic between sources and destinations on the communication graph, and interacts deeply with the virtual topology design problem [4]. There are mathematical models addressing both problems at the same time [1].

We consider a network as a multi-graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of $|\mathcal{V}|$ nodes. Each node $v \in \mathcal{V}$ corresponds to a telecommunication center. Each edge $e \in \mathcal{E}$ corresponds to a cable containing $\mathcal{F}_{(m,n)}$ optical fibers from telecommunication center m to n. The topology considered is arbitrary (mesh) and not necessary symmetrical: we can have $\mathcal{F}_{(m,n)} \neq \mathcal{F}_{(n,m)}$. Each optical fiber can transport simultaneously $\mathcal{W}$ wavelengths $l_1, \ldots, l_{\mathcal{W}}$. Each one can transport a bandwidth $\mathcal{C}$, expressed in Mbps. For technological reasons, we consider that $\mathcal{W}$ and $\mathcal{C}$ are the same on the entire network. We believe that few telecommunication operator would build an heterogeneous network. However, it is quite simple modify our model to consider heterogeneous lightpath capacity. For each pair $(s, d) \in \mathcal{V}^2$ a demand request $\mathcal{D}_{(s,d)}$, expressed in Mbps, is defined.

When defining a virtual topology, our aim is to define a set of lightpaths $\mathcal{L}$. We denote $l_w^{(i,j)}$ an elementary path on G from i to j using wavelength w of each edge supporting the lightpath. There can be many lightpaths going from a node s to a node d and they may not follow the same route on the physical network. The set of nodes $\mathcal{V}$ and the set of lightpaths $\mathcal{L}$ define a multi-graph $\mathcal{T} = (\mathcal{V}, \mathcal{L})$ corresponding to the virtual topology. The routing problem consists of finding a flow of the demands on the graph $\mathcal{T}$. We make multi-hop routing: the demands may reach their destination going through more that one lightpaths. We also allow the demands to be split.

B. MILP model

A common formulation for such problem is a source-destination flow formulation [1], in which there is a variable making the association between each commodity and each link. In our case, there is a high number of commodities going through the network. The number of generated variables and constraints is very high.

It can be reduced by aggregating all commodities from a given node. If the cost associated with each edge does not depend on the commodity, both approaches are equivalent [5]. This leads us to a source flow formulation of the VTDR problem. Such a formulation is used for the virtual topology design problem in [6]. Such formulation allows to reduce the computer memory occupation of the problem.

C. Variables

We define the following variables:

- $p^i_{(m,n),w}$ is the number of lightpaths from node i using wavelength w of link $(m,n) \in \mathcal{E}$.
- $c^{(i,j)}_w$ is the number of lightpaths from node i to node j using wavelength w.
- $c^{(i,j)}$ is the number of lightpaths from node i to node j.
- $f^s_{(i,j)}$ is the flow from source s using lightpath (i,j).

The overall number of variables is $O\left(|\mathcal{V}|^3\mathcal{W}\right)$.

D. Virtual topology design constraints

The constraints associated with the virtual topology design problem are the following:

$$\sum_{(i,n)\in\mathcal{E}}\sum_{w=1}^{\mathcal{W}} p^i_{(i,n),w} = \sum_{j\in\mathcal{V}} c^{(i,j)}, \forall i \in \mathcal{V} \quad (1)$$

$$\sum_{(m,n)\in\mathcal{E}} p^i_{(m,n),w} - \sum_{(n,p)\in\mathcal{V}} p^i_{(n,p),w} = c^{(i,n)}_w, \quad \begin{array}{l}\forall i,n \in \mathcal{V}^2\\ i \neq n\\ 1 \leqslant w \leqslant \mathcal{W}\end{array} \quad (2)$$

$$\sum_{w=1}^{\mathcal{W}} c^{(i,j)}_w = c^{(i,j)}, \forall i,j \in \mathcal{V}^2 \quad (3)$$

$$\sum_{i\in\mathcal{V}} p^i_{(m,n),w} \leqslant \mathcal{F}_{(m,n)}, \quad \begin{array}{l}\forall (m,n) \in \mathcal{E}\\ 1 \leqslant w \leqslant \mathcal{W}\end{array} \quad (4)$$

Constraints (1) corresponds to the flow conservation for each source node i. Constraints (2) corresponds to the flow conservation in destination nodes n, for each wavelengths. Constraints (3) corresponds to the number of lightpath conservation. Constraints (4) corresponds to the capacity constraints. As we consider multi-fiber networks, we have to consider such capacity wavelength by wavelength: We cannot allow twice the same wavelength in a given fiber, and consequently we cannot allow each wavelength more than there are fibers installed. The number of constraints generated for the virtual topology design problem is $O\left(|\mathcal{V}|^2\mathcal{W}\right)$.

E. Routing constraints

$$\sum_{j\in\mathcal{V}} f^s_{(s,j)} = \sum_{d\in\mathcal{V}} \mathcal{D}_{(s,d)}, \forall s \in \mathcal{V} \quad (5)$$

$$\sum_{i\in\mathcal{V}} f^s_{(i,k)} - \sum_{j\in\mathcal{V}} f^s_{(k,j)} = \mathcal{D}_{(s,k)}, \forall (s,k) \in \mathcal{V}^2, k \neq s \quad (6)$$

$$\sum_{s\in\mathcal{V}} f^s_{(i,j)} \leqslant \mathcal{C}c^{(i,j)}, \forall (i,j) \in \mathcal{V}^2 \quad (7)$$

Constraints (5) corresponds to the flow conservation constraints in source node s. Constraint (6) corresponds to flow conservation in destination node k. Finally, constraints (7) is the capacity constraint. The number of constraints generated for the routing is $O\left(|\mathcal{V}|^2\right)$.

F. Additional cuts

1) Number of lightpaths required: The sum of the demands from node s is a lower bound for the overall traffic leaving s. Similarly, the sum of the demands to node d is a lower bound for the overall traffic arriving in d. As the capacity of each lightpath is defined, this gives us a lower bound on the number of lightpaths from s (traffic leaving s - constraints (8)) and to d (traffic reaching d - constraints (9)). These cuts are commonly used for flow problems.

$$\sum_{(i,n)\in\mathcal{E}}\sum_{w=1}^{\mathcal{W}} p^i_{(i,n),w} \geqslant \left\lceil \frac{\sum_{d\in\mathcal{V}} \mathcal{D}_{(i,d)}}{\mathcal{C}} \right\rceil, \ \forall i \in \mathcal{V} \quad (8)$$

$$\sum_{i\in\mathcal{V} i\neq j} c^{(i,j)}_w \geqslant \left\lceil \frac{\sum_{s\in\mathcal{V}} \mathcal{D}_{(i,d)}}{\mathcal{C}} \right\rceil, \ \forall j \in \mathcal{V} \quad (9)$$

2) Flow and number of lightpaths: An equation linking the flow variables and the number of lightpaths can be defined. It "helps" making the $c^{(i,j)}_w$ being equal to zero if the flow variable is equal to zero [1]. This cut can be expressed this way:

$$f^s_{(i,j)} \leqslant \sum_{d\in\mathcal{V}} \mathcal{D}_{(s,d)} \sum_{w=1}^{\mathcal{W}} c^{(i,j)}_w, \ \forall (s,i,j) \in \mathcal{V}^3, s \neq j \quad (10)$$

G. Metrics

For the VTDR problem, various metrics can be used.

a) Number of wavelengths: The number of used wavelengths is a commonly used metric and represents the number of transmitters and receivers needed. It has direct influence on the cost of the switches used. We can express this metric in the following way:

$$f_1 = \sum_{i\in\mathcal{V}}\sum_{(m,n)\in\mathcal{E}}\sum_{w=1}^{\mathcal{W}} p^i_{(m,n),w} \quad (11)$$

b) Maximum link load in number of lightpaths: Minimizing the maximum link load in number of lightpaths allows to distribute the lightpaths between all the links. That avoids having a small set of links carrying all lightpaths. Well-distributed lightpaths make network evolution and management more flexible, since some capacity remains available in all links. It allows to perform easily load balancing, to allocate dedicated protection paths, and so on. Let us call $f_2 = M_l$ the maximum link load. We need to include the following constraint:

$$\sum_{i \in \mathcal{V}} \sum_{w=1}^{\mathcal{W}} p^i_{(m,n),w} \leqslant M_l, \ \forall\, (m,n) \in \mathcal{E} \tag{12}$$

c) Average number of hops: The average number of hops has a direct influence on the transmission time [1], [7]. In our model, we consider that a signal goes through electronic devices only when it enters or leaves a lightpath. Such operation is considered as “slow”, as it requires optical-electronic conversions. The number of hops of a demand from s to d is the number of lightpaths that the signal goes through. We can express this metric with the following constraint.

$$f_3 = \frac{1}{\sum_{s \in \mathcal{V}} \sum_{d \in \mathcal{V}} \mathcal{D}_{(s,d)}} \sum_{s \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} f^s_{(i,j)} \tag{13}$$

H. Integrality constraints

Some of the variables considered have to be integer:

- $p^i_{(m,n),w} \in \mathbb{N}$
- $c^{(i,j)}_w \in \mathbb{N}$
- $c^{(i,j)} \in \mathbb{N}$

However, we can relax this integrality constraint for some variables. $c^{(i,j)}_w$ will necessary be integer since they are the sum of integer variables. This is also the case of $c^{(i,j)}$. Doing so, the number of integer variables is $O\left(|\mathcal{V}|^3\mathcal{W}\right)$, and the number of continuous variables is $O\left(\left(|\mathcal{V}^2|\mathcal{W} + |\mathcal{V}|^3\right)\right)$.

III. Determination of the Pareto frontier

A. The Pareto frontier

We selected three metrics. Each one gives interesting informations about the performance of the network. To have an idea of the relationships between these metrics, we have to carry out a multiobjective analysis of the problem. This means that we will search for the Pareto frontier of the problem [8]. It is a set of “best” solutions.

We define our multiobjective optimization problem in the following way:

$$\left\{ \begin{array}{l} \min_x F(x) \\ x \in \mathcal{P} \end{array} \right. \qquad F = \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix} \tag{14}$$

where $\mathcal{P}$ is the set of feasible solutions. We denote $F_i(x)$ the i^{th} component of $F(x)$.

The objective function of our optimization problem is in $\mathbb{R}^m$. There are probably no solution x^* such that $\forall x \in \mathcal{P}, F(x^*) \leqslant F(x)$. Note that if such solution exists, it has to be chosen by the decision maker, since there is no solution performing better with any metric[1]. We say that a point $F(x) \in \mathbb{R}^m$ *dominates* a point $F(y) \in \mathbb{R}^m$ if $\forall i, F_i(x) \leqslant F_i(y)$ and $\exists j / F_j(x) < F_j(y)$. The Pareto frontier is the set of non-dominated points. Informally speaking, a point belongs to the Pareto frontier if and only if the only way to improve the performance with one metric is to decrease the performance with other metrics.

B. Methodology

The problem considered has the following characteristics: The Pareto frontier is discrete, as the set of feasible solutions of the problem is discrete. The number of constraints and variables generated by an instance of our problem is high. The use of non-linear constraints would increase the difficulty of solving the problem.

The method used to search for the Pareto frontier have to take into account those facts. We based our method on an $\epsilon-$restricted method [8].

$\epsilon-$restricted method corresponds in solving problems of the following format, called $\epsilon-$problems:

$$\left\{ \begin{array}{l} \min_x f_i \\ x \in \mathcal{P} \\ f_j \leqslant \epsilon_j ; j \neq i \end{array} \right. \tag{15}$$

Figure 1 illustrates the key idea of the $\epsilon-$method: minimizing f_1 will return F^*_1. If the restriction $f_2(x) \leqslant \epsilon_2$ is added to the problem, minimizing f_1 will not return F^*_1, but one of the points of the Pareto frontier.

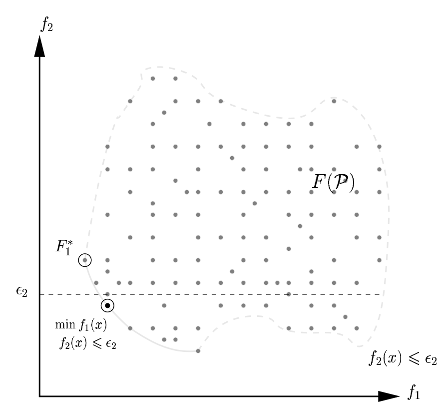

Fig. 1. ϵ based method minimizing f_1

We first need to solve each mono-objective problem to get f^*_i the value of the optimal solution for metric f_i. F^* is the vector of all those best values (ideal point). We then build the vector $\dot{F}$, vector of worst values reached for each objective. This point is called *Nadir point*. With F^* and $\dot{F}$, we know the possible variation, for each metric, of the points belonging to the Pareto frontier: each point $F(x)$ of the Pareto frontier necessary verify $F^* \leqslant F(x) \leqslant \dot{F}$ (see figure 2).

To obtain points belonging to the Pareto frontier, we solve $\epsilon-$problems (15). We choose the ϵ_j such that $f^*_j \leqslant \epsilon_j \leqslant$

[1]Such a solution is called *ideal point*

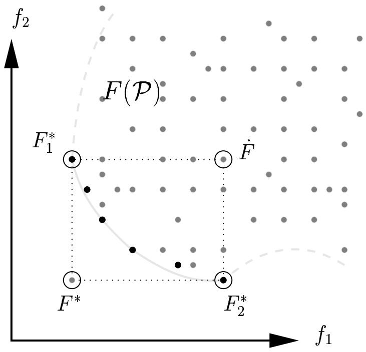

Fig. 2. Pareto set for combinatorial optimization

$\dot{F}_j$. We solve the associated problem to get a point of the Pareto frontier. It is worth noting that if we consider three or more metric at the same time, the $\epsilon-$problem generated can be empty.

IV. Experiments and Results

We applied this method for finding the Pareto frontier of three networks: two small networks (6 nodes) and a medium network (11 nodes). Those networks are represented on figures 3, 4 and 5. For all network, the traffic matrix has been randomly generated.

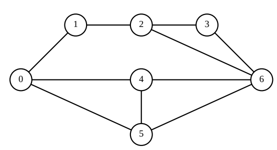

Fig. 3. Small network 1

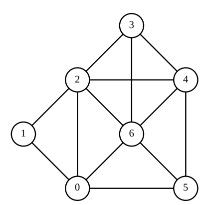

Fig. 4. Small network 2

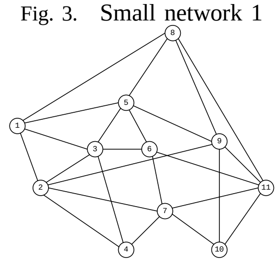

Fig. 5. Medium network

Both mono-objective and $\epsilon-$problems have been solved using the commercial software `Cplex`[2], on PC platform.

A. First small network

For each pair (s, d), the demand request $\mathcal{D}_{(s,d)}$ is randomly and independently chosen between 0 and 200 Gbps. The others parameter are the following: $\forall\,(i,j) \in \mathcal{E}, \mathcal{F}_{(i,j)} = 20$. $\mathcal{W} = 8$ and $\mathcal{C} = 40$Gbps.

We first focus on two metrics at the same time. We compute the Pareto frontier of objectives (f_1, f_2). The ϵ_j have been randomly generated, such that $f_j^* \leqslant \epsilon_j \leqslant \dot{F}_j$. For each $\epsilon-$vector, we generate a $\epsilon-$problem that we solve, giving us a point of the Pareto frontier. The results are represented on figure 6.

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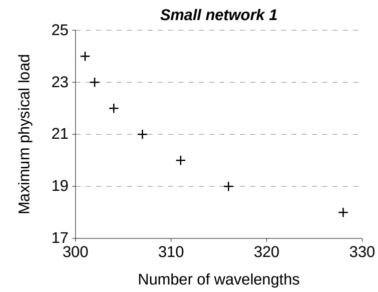

Fig. 6. Pareto frontier (f_1, f_2)

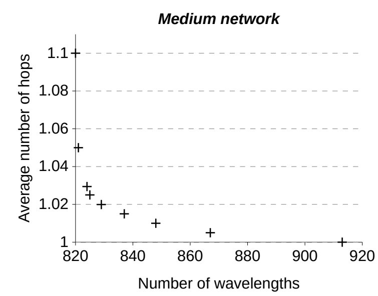

Fig. 7. Pareto frontier (f_1, f_3)

We then consider the three metrics at the same time. We also generate the $\epsilon-$vectors randomly. However, many generated problems turn out to be empty. The following table gives the Pareto points that we obtained.

Wavelengths	*Phys. load*	*Hops*	*Wavelengths*	*Phys. load*	*Hops*
330	18	1.189592	303	24	1.067792
329	26	1.000000	311	25	1.032060
303	25	1.055696	301	24	1.099203

B. Medium network

For each pair (s, d), the demand request $\mathcal{D}_{(s,d)}$ is randomly and independently chosen between 20 and 150 Gbps. The other parameters are the following: $\forall\,(i,j) \in \mathcal{E}, \mathcal{F}_{(i,j)} = 3$. $\mathcal{W} = 32$ and $\mathcal{C} = 20$Gbps.

We restrict ourselves to two objective functions. In the experiments made with the first small network, we choose randomly the $\epsilon-$vectors. We make many experiments and we obtain many times the same solution. In these experiments, we choose the $\epsilon-$vectors "manually" in order to cover the whole domain. This allowed us to avoid useless computation and to guide the exploration of the Pareto frontier. The obtained results are represented on figure 7.

C. Second small network

For each pair (s, d), the demand request $\mathcal{D}_{(s,d)}$ is randomly and independently chosen between 20 and 150 Gbps. The other parameters are the following: $\forall\,(i,j) \in \mathcal{E}, \mathcal{F}_{(i,j)} = 4$. $\mathcal{W} = 32$ and $\mathcal{C} = 20$Gbps. Again, we chose the $\epsilon-$vectors to explore more smartly the Pareto frontier.

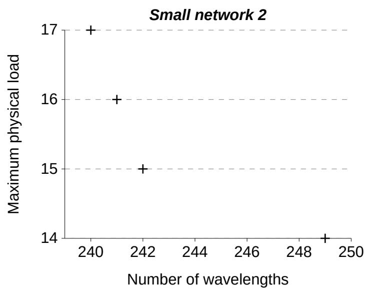

Fig. 8. Pareto frontier (f_1, f_2)

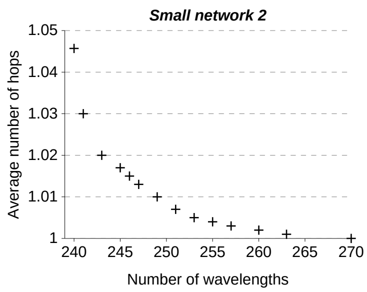

Fig. 9. Pareto frontier (f_1, f_3)

D. Results analysis

The first small network has a diameter higher than the second small network. In this case, decreasing the maximum link load is not compatible with decreasing the number of wavelengths used; since some links carry almost all the short paths. With the second small network, which is almost a complete graph, the trade-off is not so strong.

For the second small network, we were able to map completely the Pareto frontier: The value of the maximum physical load has to be integer. We obtained a solution for each possible value of the maximum link load.

The more visible trade-off between the metrics is the number of wavelengths against the average number of hops. This is intuitive: to get a low number of hops, it is required to add dedicated paths for all demands, which increases the use of network resources. But our multiobjective approach, allows us to quantify this trade-off.

Choosing randomly the values of the ϵ_i is quite simple. However, letting the decision maker (or any other clever decision process) choose these values is better: it avoids unnecessary (the results obtained are sufficient) or useless computation (solving two $\epsilon-$problems almost equal, giving the same point of the Pareto frontier.

V. Conclusion and future works

We applied multiobjective optimization techniques to a classical telecommunication network problem. We present the mathematical model, different metrics, and a method to identify the Pareto frontier. Instead of requiring the best solution for a given metric, leaving a small margin with a metric allows to improve greatly the quality of the solution in relation with other metrics. This also justifies the interest of performing such multiobjective analysis.

The method proposed needs to be improved. However, we believe that such an approach gives the decision maker valuable data about the way how he can configure the network. We are currently performing more tests and working with reoptimization techniques to improve the overall efficiency of our method.

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