Multiobjective Analysis in Wireless Mesh Networks

Cristiana Gomes, Gurvan Huiban
INRIA/I3S(CNRS-UNIVERSITÉ DE NICE SOPHIA ANTIPOLIS)
2004 route des lucioles - BP 93 FR-06902 Sophia Antipolis, France
{cristiana.gomes,gurvan.huiban}@sophia.inria.fr

***Abstract*— Wireless Mesh Networks are a scalable and cost-effective solution for next-generation wireless networking. In the present work, we consider the *Round Weighting Problem (RWP)* which solves a joint routing and scheduling problem to satisfy a given demand subjected to the multi-access interferences.**

We propose a multiobjective approach that deals with two objectives. The first one is to balance the load in the routers, increasing the security in case of failure. The second objective is to minimize the communication time. We aim to find the Pareto set of the problem, as it captures the trade-off generated by using these two conflicting objective functions.

We make experiments with some networks with different number of gateways. The column generation method is used to solve efficiently the test instances. Our approach finds out the relationship between the objective functions, corresponding to a convex piecewise linear function.

I. INTRODUCTION

There is an increasing interest in using *Wireless Mesh Networks* (*WMNs*) as broadband backbone for next-generation wireless networking. A WMN is a scalable and cost-effective solution. Mesh networking, in which information is routed from origin to destination over multiple wireless links, has potential advantages over traditional single-hop networking, especially for back-haul communication [1].

WMNs are composed of wireless mesh *routers* and *clients*. Wireless mesh routers, working as access points, constitute a multihop wireless network that serves as backbone providing network access for the mesh clients. A special kind of routers, called *gateway*, interfaces with other networks as illustrated in figure 1. The wireless mesh routers are usually stationary [2].

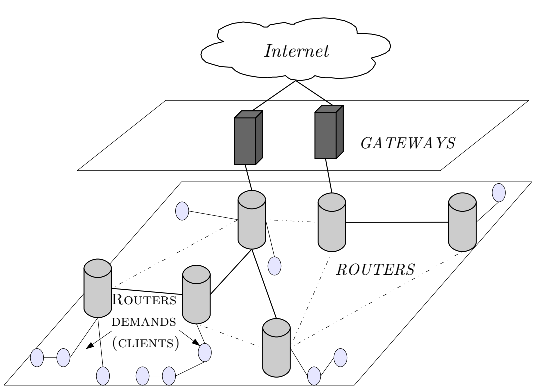


Fig. 1. Wireless Mesh Network.

In wireless networks, the communication channels are shared by the terminals. Thus, one of the major problems faced is the reduction of capacity due to interferences caused by simultaneous transmissions [3]. In this work, we call a *round* a collection of links that can be simultaneously activated in the network.

We focus on a joint routing and scheduling problem in wireless networks subjected to the multiaccess interferences, so called *Round Weighting Problem* (*RWP*) [4]. A *column generation* approach is used to select the rounds improving the objective function, avoiding the generation of the whole set of rounds that is exponential.

We propose a multiobjective approach for the *RWP*, considering two objectives. The first one is to balance the load in the routers (*MinMaxLoad*). It increases the security in case of failure. The second objective is to minimize the communication time (*MinTime*). It corresponds to the time required to route all router demands.

Multiobjective optimization does not compute an unique solution, but a set of "best" solutions, called the *Pareto optimal set*, capturing the trade-offs between the different metrics. Solving a multiobjective problem consists in finding the Pareto optimal set, from which the decision maker choose the solution that fits the best his needs. In this work, each point of the Pareto set is obtained by solving an optimization problem.

The main contribution of this work is to give a multicriteria vision of the Round Weighting Problem. As far as we know there is no multiobjective analysis in this subject. The model in this article can be useful as a benchmark for networks with distributed scheduling, like in IEEE 802.11s. It can also be useful in a context where centralized scheduling can be adopted, like in IEEE 802.16d, that can directly take advantage of our analysis. These IEEE standards are specific to wireless mesh networks.

This paper is organized as follows. In the next section, we discuss the related works. In section III, we present the *RWP* formulation, its decomposition for the column generation method and our multiobjective approach with ϵ-restricted technique. In section IV are presented some of the experimental results we obtained. We conclude the paper and give the future directions in section V.

II. RELATED WORKS

The joint routing and scheduling optimization in the WMNs is a recent topic. A key issue in wireless networks is the interferences produced between neighboring routers. Interference models have been introduced using either conflict graphs or signal to noise ratio [5]. Their impact on shortest path routing has been investigated in [6] and [7].

In order to deal with interferences, it is important to know what are the sets of transmissions that can be active at the same time (the rounds). The Round Weighting Problem was treated in [4] with the objective to minimize the round number. The authors make dual analysis and propose approximation algorithms for some specific graphs. They show the NP-hardness of this problem by proving that the well-known NP-hard problem of finding the *Fractional Coloring* on unit graphs reduces to it. The Fractional Coloring was proved NP-hard by [8] and [9].

An algorithm enumerating a tractably large subset of simultaneous transmission rounds has been developed in order to compute an approximated solution for maximum throughput using linear programming (LP) in [10]. Solving the full LP problem means generating an exponential set of scenarios which is intractable even for small networks. Several works use column generation method as in [11] and [12]. In [12], this method is associated with set covering formulations to model the resource allocation problem in ad-hoc radio networks.

III. Hypotheses and problem definition

In this section we give some definitions that will help to understand the problem. The *RWP* can be modeled as a graph problem. A wireless topology is represented as a digraph $G = (V, E)$. The set of routers is represented by the set of nodes V. The set of edges $E \subseteq V \times V$ corresponds to the communication links from the real network. If a router j is located within the transmission range tr_i of a router i, considering range distance, obstacles, etc, then there is an edge $(i, j) \in E$.

We consider the link (i, j) active when the router i is transmitting data to j. In this case, it interferes with another links located within the interference range it_i of router i. The set $I_{u,v}$ is composed by all links interfering with the link (u, v). Consequently flexible binary interference models can be adopted.

A round in a wireless network corresponds to a set of links that can be active at the same time without making interferences between them. The size of the complete set of rounds is exponential. We consider a column generation approach to select as required the rounds to improve the solution of the problem. The round definition guarantees that the communication will be multiaccess interferences free in G.

We focus on router-gateway traffic pattern, naturally modeled by a multicommodity flow problem. The commodities are going from the set of nodes V_r to the set of gateways V_g ($V_r \cup V_g = V$ and $V_r \cap V_g = \emptyset$).

Given a graph $G(V_r \cup V_g, E)$, a set of router demands d_v with $v \in V_r$ and an interference model, the *Round Weighting Problem (RWP)* is to find the set of rounds to satisfy the given demand. From this set of rounds can be deduced the paths followed by the data. We deal with two objectives: *MinMaxLoad* and *MinTime*. In *MinMaxLoad* we try to balance the load in the routers and in *MinTime* the goal is to minimize the communication time. More precise definitions of the objective functions will be given in section III-B.

A. Column Generation Method

The problems considered are the RWP+*MinMaxLoad* and the RWP+*MinTime* taking into account the complete set of rounds. As the number of rounds is exponential, the number of columns of the constraint matrix is exponential. The key idea of the column generation is that it is not needed to list explicitly all of the columns of the problem formulation, but rather to generate them only “as required” [13]. The problem is decomposed into a master problem and a sub-problem. We solve the master problem with a small subset of columns, which serves as an initial basis. The sub-problem is then solved to check the optimality of the solution under the current master basis and to generate new columns for the master problem. This procedure repeats until the master problem contains all columns necessary to find the optimal solution of the original problem. Each column corresponds to one round.

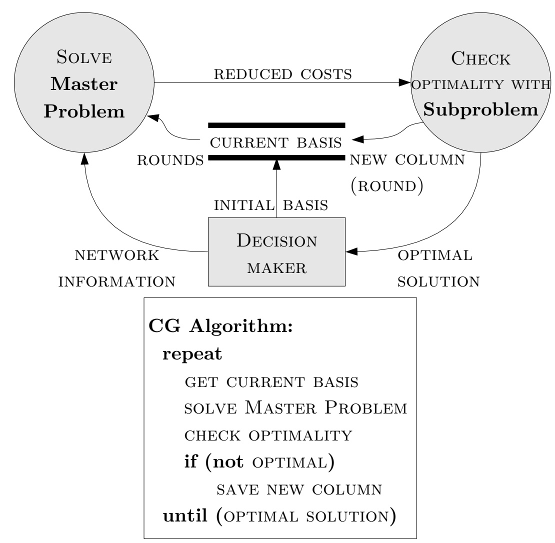


Fig. 2. Column generation algorithm and data flow diagram.

In each iteration, if the sub-problem can find a new column that may improve the master solution, this column is inserted in the master basis and a new master solution is computed. If the sub-problem cannot find a new column, it means that the solution of the problem is optimal. This algorithm is represented on figure 2. This column generation approach is close to the one proposed by Gilmore and Gomory [14]. The notation and the decomposition of the problem in master model and sub-model are based in [11]. We adapted the master model to the WMNs context.

1) Master problem formulation: We define the following variables: Let the variable $x^v_{i,j}$ denotes the flow from the router v over link i, j. The demand from each router v is represented by the parameter d_v. Let the binary parameter $a^r_{i,j}$ be 1 if link (i, j) is active in the round r, and 0 otherwise.

Recall that set $I_{u,v}$ is composed by all links interfering with (u,v). We define $F_{(u,v)}^{(i,j)} = 0$ if $(i,j) \in I_{u,v}$ and 1, otherwise. We define w_r as the fraction of time that round $r \in R$ is scheduled. Consequently, there is an induced edges capacity $c_{i,j} = \sum_{r \in R} a_{i,j}^r w_r, \forall\, (i,j) \in E$.

The master problem can be defined as follow: Given a graph $G(V_r \cup V_g, E)$, a set of routers demand d_v with $v \in V_r$ and a set of rounds R, the problem is to assign a weight w_r to each round $r \in R$. The weights represent the amount of time a round will be activated. The total amount of time needed to satisfy all demand will be $w_R = \sum_{r \in R} w(r)$. From the edges of the rounds can be deduced the paths followed by the data as illustrated in figure 3. It may happen that some of the rounds r have a weight equal to zero. The load in each router $i \in V_r$ is given by $l_i = \sum_{v \in V_r} \sum_{j \in V/(i,j) \in E} x_{i,j}^v$.

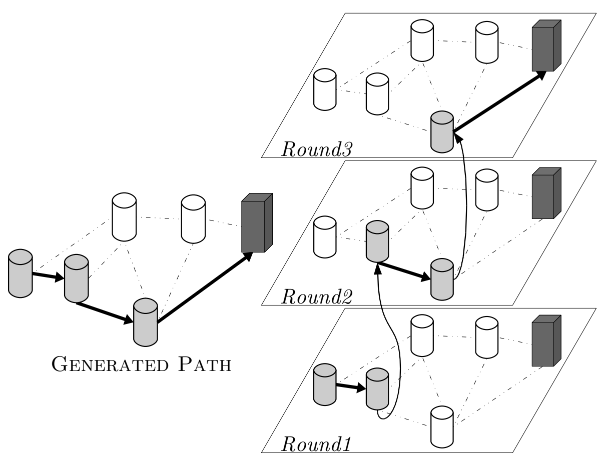


Fig. 3. Path deduction from a set of rounds.

The constraints of the master problem expressed as a linear programming model are the following:

$$\sum_{i \in V/(v,i) \in E} x_{v,i}^v = d_v, \forall v \in V_r \quad (1)$$

$$\sum_{j \in V_g} \sum_{i \in V_r/(i,j) \in E} x_{i,j}^v = d_v, \forall v \in V_r \quad (2)$$

$$\sum_{i \in V_r/(i,j) \in E} x_{i,j}^v - \sum_{k \in V/(j,k) \in E} x_{j,k}^v = 0, \forall j, v \in V_r \quad (3)$$

$$\sum_{r \in R} a_{i,j}^r . w_r - \sum_{v \in V_r} x_{i,j}^v \geqslant 0, \forall i, j \in E \quad (4)$$

Constraints (1-3) correspond to the flow constraints. Constraints (1) define the flow leaving its source router and constraints (2) define the flow arriving in a gateway. Constraints (3) represent the flow conservation, that is, the flow entering an intermediate router equals the flow leaving that router. Constraints (4) assign weights to the rounds to satisfy the flow in the edges.

2) Sub-problem formulation: The sub-problem generates a round r with the minimal *reduced cost* $\left(1 - \sum_{(i,j) \in E} p_{(i,j)} . a_{i,j}^r\right)$ to enter the master basis. To express the sub-problem as a linear programming model, we have to define some additional notations. Let the parameter $p_{(i,j)}$ be given by the dual variable associated with the constraints (4) of the master problem. Consider the binary variable $u_{(i,j)} = 1$ indicating if the edge (i,j) enters the round to be added to R, $u_{(i,j)} = 0$ otherwise. The sub-problem can be seen as the *Maximum Weighted Independent Set Problem* which is NP-hard [15]. The parameter $p_{(i,j)}$ corresponds to the weight of the edges. The objective function is to maximize the sum of the weights of all active edges respecting interferences.

The formulation of the sub-problem is the following:

$$\max \sum_{(i,j) \in E} p_{(i,j)} u_{(i,j)} \quad (5)$$

$$u_{(i,j)} + u_{(k,l)} \leqslant 1 + F_{(i,j)}^{(k,l)}, \forall\, (i,j) \in E, \forall\, (k,l) \in E \quad (6)$$

The objective function (5) searches the maximum weight, which is equivalent to the minimum reduced cost. The parameter $p_{(i,j)}$ guides the column generation to select the best round. Constraints (6) avoid interferences according to the interference model in F.

If the value of the objective function in the sub-problem is strictly greater than 1 (e.g. the reduced cost is negative), a new column $u_{(i,j)}$ is found and the master basis is expanded. Otherwise, the master problem already gives the optimal solution to the original problem.

B. Multiobjective Formulation

To evaluate the overall quality of our solutions, we use the following metrics:

- *MinMaxLoad* (f^1): Balancing the quantity of flow in the routers. The rounds are chosen in a way to minimize the maximum load l_v in the routers V_r.
- *MinTime* (f^2): Minimizing the time of the communication. It chooses the rounds in a way that the round activations time will be minimum, that is, it minimizes the total weight w_R of the schedule.

The objective function of the master problem with objective *MinMaxLoad* and *MinTime* are the following, respectively

$$\min(f^1 = \max_{v \in V_r}(l_v)) \quad (7)$$

$$\min(f^2 = w_R) \quad (8)$$

To study the trade-offs between these two metrics, we consider the problem as a multiobjective one. The main idea of multiobjective optimization is to find out all the possible *non-dominated* solutions of an optimization problem. A solution is *dominated* if there is another solution improving simultaneously all the metrics. A solution is non-dominated if there is no other solution dominating it. Informally speaking, it means that if a solution is non-dominated within the whole solution space, it is not possible to improve one of the metrics without worsening at least one of the other metrics. The set of all non-dominated solutions is the *Pareto set* [16].

In multiobjective optimization, the solution space is a part of R^m where m is the number of metrics. In our case $m = 2$. The optimization is performed on the plane and as there is no total order relation in R^2, there is not a single but many "best solutions".

A multiobjective optimization problem can be defined in the following way:

$$\bar{F} = \begin{cases} \min_x F(x) \\ x \in \mathcal{P} \end{cases} \quad \text{where } F : \begin{cases} \mathcal{P} & \rightarrow & R^2 \\ x & \mapsto & \begin{pmatrix} f^1(x) \\ f^2(x) \end{pmatrix} \end{cases} \tag{9}$$

$\mathcal{P}$ is the set of feasible solutions, defined by constraints (1) to (4).

C. ϵ-restricted technique

The idea of the ϵ-restricted technique is to add additional constraints preventing the solver to return one of the optimum solution of one of the induced mono-objective problems, as described in [17] and [18]. More precisely, the $\epsilon-$restricted technique corresponds to generating and solving mono-objective problems under the form:

$$\begin{cases} \min_x f^i \\ x \in \mathcal{P} \\ f^j \leqslant \epsilon^j; j \neq i \end{cases} \tag{10}$$

The ϵ^i are chosen such that $\bar{f}^i \leqslant \epsilon^i$, where $\bar{f}^i$ corresponds to the optimum value of the mono-objective problem minimizing objective f^i. Figure 4 illustrates the key idea of the $\epsilon-$method: Solving the classical mono-objective problem minimizing f^1 gives $\bar{f}^1$. If the restriction $f^2(x) \leqslant \epsilon^2$ is added to the problem, minimizing f^1 will not return $\bar{f}^1$ anymore but another points of the Pareto optimal set. The same can be applied when minimizing f^2.

The Pareto set provides to the decision maker the trade-offs, allowing him to choose the solution that he considers as the best one.

IV. Results

The model was coded using the AMPL modeling language and it was solved using the commercial software Cplex version 10, on a desktop PC with one gigabyte of RAM. We used the mesh networks instances from [19]. We defined a simple interference model where each edge interferes with another one if the distance between them in graph G is lower than 2.

We represent some of the obtained results on figures 5 to 7. The results are represented in the solution space: The x axis represents the communication time, and the y axis represents the maximum load. Each point corresponds to a solution.

This approach using column generation and multiobjective optimization appears to be quite efficient, as the computation time to solve any instance is low, of the order of tenths of seconds. The overall time f^1 as well as the maximum load f^2 decrease as the number of gateways increases.

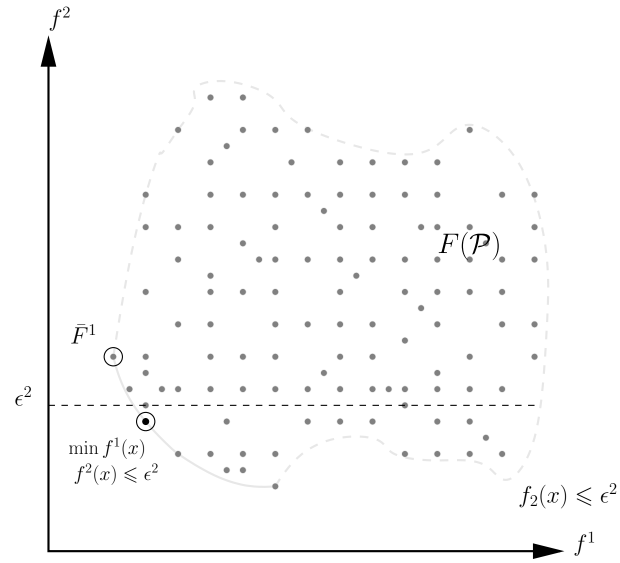
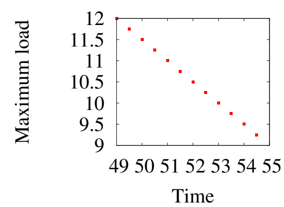
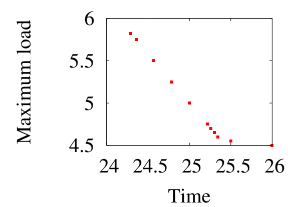


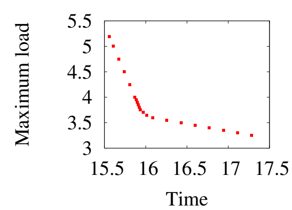
Fig. 4. ϵ based method minimizing f^1



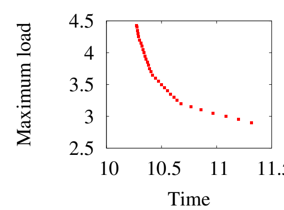
(a) 1 gateway



(b) 2 gateways

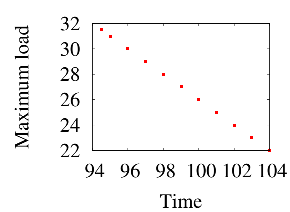


(c) 3 gateways

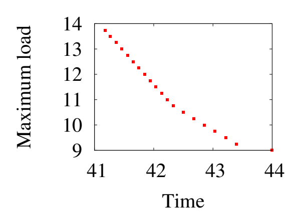


(d) 5 gateways

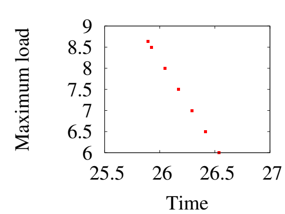
Fig. 5. 39 nodes mesh network (giul69 instance)



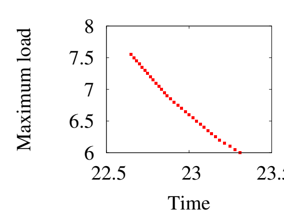
(a) 1 gateway



(b) 2 gateways

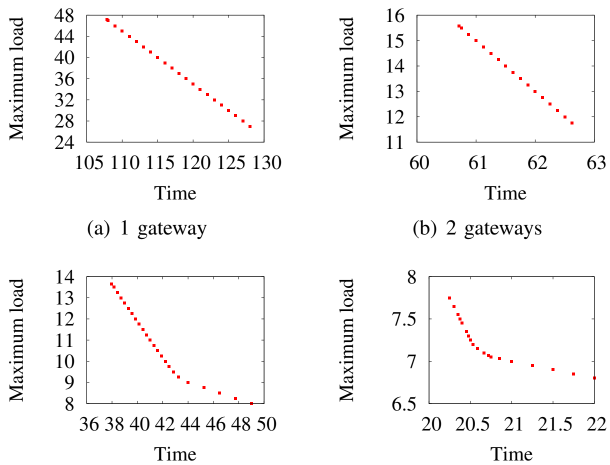


(c) 3 gateways



(d) 5 gateways

Fig. 6. 65 nodes mesh network (ta2_65 instance)



(a) 1 gateway (b) 2 gateways

(c) 3 gateways (d) 5 gateways

Fig. 7. 54 nodes mesh network (zib54 instance)

As it was expected, the routing generates bottlenecks located around the gateway(s), because all the flow goes toward them. We observe that when the routing use distinct paths to route the flow, it allows to activate different edges in the same round, reducing the overall transmission time. Informally speaking, it may be more efficient to follow different routes that do not interfere one with another than following shorter routes resulting in more interferences.

Minimizing the time increases the maximum load of the routers. We observe that the relation between the maximum load and the transmission time seems convex and piecewise linear. The linear parts corresponds to the following situations: As we make tighter the value of the maximum load, an amount of flow is deported on another path. Using this other path results in an increase of the overall transmission time. Hence, for each unit of flow following the second path, the overall transmission time increases by a given value (the difference of time between the first path and the second one).

Each disruption in the graphs is due to the happening of a new bottleneck, forcing a flow transfer on a path that is not the best possibility. It may activate some path that was not in use. As a consequence, the rate of time per flow increases.

A disruption situation is illustrated on figure 8, where routers 2,4 and 5 send data to gateway 1. We assume that each router has only one unit of traffic to send, $d_v = 1$. For clarity reason, we consider that the time unit is the second (s) and the flow unit is the Gigabit (Gb), even if the formulation is independent from the unit chosen. Let us consider only the flow from router 4, because routers 2 and 5 send directly their flow to the gateway. The flow from router 4 follows three different paths $p_1 = 4-5-1, p_2 = 4-3-2-1$ and $p_3 = 4-3-7-8-6-1$ to reach the gateway. When the maximum authorized load is 1.25Gb, the flow from router 4 is divided the following way: 0.25Gb follow p_1, 0.17Gb follow p_2 and 0.58Gb follow p_3. Router 5 is the only bottleneck of the network. With a tighter maximum load of 1.2Gb, 0.05Gb of flow are deported from path p_1 to paths p_2 and p_3, resulting in an increase of required time of 0.02s, that is, with a rate of $-0.05/0.02 = -2.5$Gb/s. But now there are two bottlenecks: routers 2 and 5. With a tighter maximum load of 1.15Gb, flow from paths p_1 and p_2 are deported to the path p_3, resulting in an increase of required time of 0.1s, that is, with a rate of $-0.05/0.1 = -0.5$Gb/s. The overall results obtained with this example are represented on figure 9.

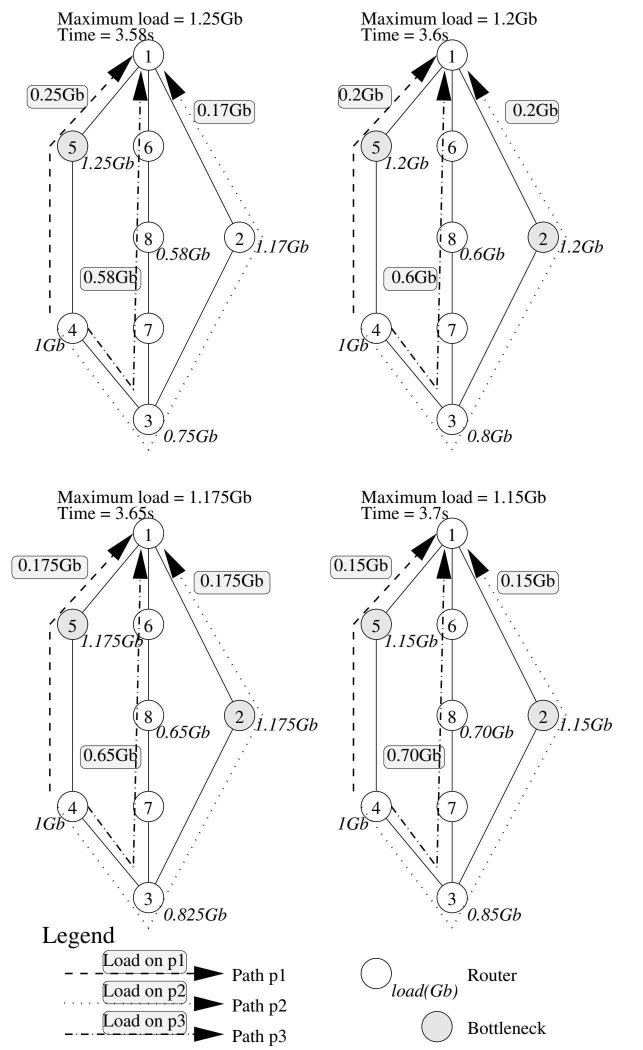


Fig. 8. Small example for piecewise linear functions.

V. Conclusion and Perspectives

In this article, we solve the Round Weighting Problem, which corresponds to a joint routing and scheduling problem to satisfy a given demand subjected to the multiaccess interferences. We propose a multiobjective approach relating the overall transmission time, expressed in number of rounds, and the maximum load. The problem is solved using a column generation approach.

We make experiments with some networks with different number of gateways. The multiobjective approach allows us to obtain results about the relationship between the maximum

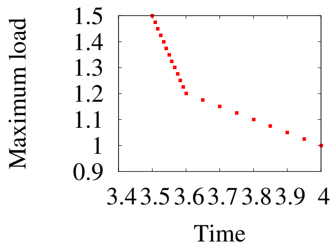


Fig. 9. Result obtained with the example from figure 8.

load and the overall transmission time. This relationship corresponds to a convex piecewise linear function. Each linear parts corresponds to the increase of time resulting by transferring part of the load from a path to another. Each disruption is due to the happening of a new bottleneck, forcing a flow transfer from a path to another.

We are currently working on improvements to the mathematical formulation, to better model wireless mesh networks. We are taking the multiobjective analysis further and also making experiments with other network topologies. Our next step will be to work with a branch and price approach. We aim for investigate whether the round-up property holds for the RWP and we also look for interesting cuts for the model.

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