A MILP model for the reconfiguration problem in multi-fiber WDM networks

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Abstract. *We address the reconfiguration problem in multi-fiber WDM networks. It consists of finding out which adaptations should be made to the virtual topology and the routing when the traffic evolves. We propose a Mixed Integer Linear Programming (MILP) model solving the problem for different objective functions. We tried to make a concise model in relations with the number of variables and restrictions, to reduce the memory occupation during the optimization process. We also add some cuts to the model.*
We make some experiments with this model and compare the results obtained with a simple greedy algorithm and with an algorithm from the literature.

1. Introduction

WDM (Wavelength Division Multiplexing) revolutionized the use of optical technology for data transmission. It allows to transmit at the same time various signals in a unique fiber by using different wavelengths, as long as they are all different one from another within an optical fiber. Nowadays, commercial equipments are able to have an overall throughput greater than one terabits per second (Tbps). In laboratory, throughput of 5Tbps has been reached [Télécom, 2002b].

With WDM technology, network architecture is no more restricted to point-to-point connections. There is an optical layer responsible for the transmission of the signal [Ramaswami and Sivarajan, 1998]. Within this layer, no electronic processing is performed. Such processing requires an optical-to-electronic conversion, which delays the signal. It also depends on the signal modulation. Finally electronic devices that are able to process signals at a very high bit rates are expensive.

1.1. Virtual topology

The optical layer is constituted by lightpaths. A lightpath is a connection between a pair of network nodes. It can be direct (point to point) or indirect (the lightpath goes across a succession of intermediate nodes). From a logical point of view, a lightpath from node A to node B represents an indivisible link, independently from going across intermediate nodes or not. The set of lightpaths is called the virtual topology or logical topology. This is illustrated on figure 1, which shows a physical topology and how the lightpaths are defined, and figure 2 which shows the resulting virtual topology.
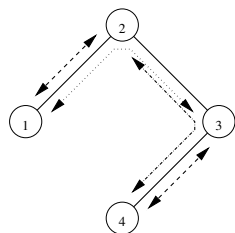

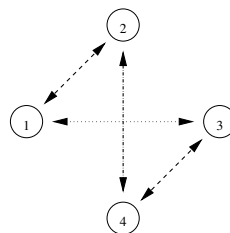**Figure 1: Physical topology and a set of lightpaths**

Figure 2: The associated logical topology.

Data to be sent from node 2 to node 3, will go through node 4, even though there exists a direct link between node 2 and 3 on the physical topology.

Developing an optical layer allows to turn two nodes, non-adjacent in the physical topology, into two adjacent nodes from a logical point of view. Data using a logical direct link will not suffer from optical-to-electrical conversion.

A single optical fiber cannot carry simultaneously two lightpaths using the same wavelength. As we decided to enforce the wavelength continuity constraint, the wavelengths are the same all along the lightpath. Technically, if we want to use different wavelengths along a lightpath, we have to add a converter, which is an expensive device [Beauquier, 2000]. As optical converters are still prototypes and are not commercially available, optical-to-electrical conversions are required, which introduces delay.

The virtual topology design problem can be defined as: given a physical topology, what is the set of lightpaths giving the best performance for a given metric? This problem is NP-hard and it is equivalent to the n-graph coloring problem [Chlamtac et al., 1992].

1.2. Routing

For a given network, we have an amount of data to send from a node to another. The routing problem consists of defining which lightpaths will be used to transmit those data, optimizing a performance criterion. This problem can be reduced to a flow formulation. The complexity of such problems is polynomial as long as the flows considered can be described by continuous variables. When we consider integer flow formulation, the problem becomes NP-hard.

As we can see, the relationship between the virtual topology and the routing problems are very strong, and it is common to consider the two problems simultaneously.

1.3. The VTDR problem

We call by VTDR (Virtual Topology Design and Routing) problem the union of the virtual topology design and routing problems. This is one of the key problems in the design of a WDM network, and it has been widely studied in the literature. As it is an extension of the virtual topology design problem and it is NP-hard.

Many variations exist for this problem involving many parameters such as topology (ring, grid, mesh), technical details (single-hop or multi-hop, with or without wavelength converters), used metrics, and so on.

A common mathematical formulation for this problem is based on multicommodity flow for lightpath and traffic flow for virtual topology [Banerjee, 1996]. However, due to the problem complexity, such approach is not suitable for medium size or large networks. A survey concerning this problem can be found in [Dutta and Rouskas, 1999].

1.4. Connection oriented communication

The bandwidth offered by optical technology is very high, and one limitation comes from the electronic part of the transmission scheme. The devices that are able to deal with tenths of gigabits per seconds are very expensive. To reduce the amount of electronic processing, we can choose to make connection oriented communications. Circuits are established and routes are defined *a priori*.

There are researches currently performed in order to implement directly on the optical layer an IP layer. However, this would require being able to perform optical routing, and this is not feasible technologically in the short term.

1.5. Multi-fiber networks

Installing a large-scale telecommunication network is something expensive. For instance, the cost of a North-American network, covering 15 cities, was estimated to 200 millions dollars [Télécom, 2002a]. An important part of the expenses comes from the substructure : digging and installing cables. Consequently, companies generally install many optical fibers at the same time, even if it is not required. This allows them to offer other services (rent dedicated lines, sell bandwidth, defining protection schemes, and so on). They can also face an increase of the use of the network.

Considering a multi-fiber network (for instance 10 fibers of 40 wavelengths) instead of a mono-fiber network offering a very high number of wavelengths available per fiber (for instance 1 fiber of 400 wavelengths - which does not exist) has consequences. It modifies the structure of some optimization problems, such as the virtual topology problem (see section 1.1). This problem is in most cases NP-hard if we consider mono-fiber networks. The multi-fiber problem turns out to be polynomial for star network, but remains NP-hard for ring networks[Li and Simha, 2001].

In section 2, we will describe the reconfiguration problem. We will then describe the MILP model we used to solve in section 3. In section 4 we describe briefly some additional cuts. In section 5 we describe some experiments we made and we finally conclude in section 6.

2. The reconfiguration problem

2.1. Traffic evolution

It has been noted that bandwidth requirements evolve with time, both in short-term and in long-term. During working hours and working days, the traffic is generally higher than during the night or week-end. The emergence of new kinds of applications (multimedia diffusion, voice over IP, etc) and the decreasing price of broadband access cause the traffic to increase continuously on the long-term.

There are various works studying network traffic. However, There is no widespread agreement about the characteristics of this traffic, due to the large amount of data flowing on backbones and the difficulty to collect them. Some models consider that traffic matrix separated by a given time are not correlated and randomly generated (see for instance [Narula-Tam and Modiano, 2000, Baldine and Rouskas, 1999b]). While other models try to simulate different kind of situations by considering a day as a set of working hours, leisure hours, night hours, . . . One interesting work can be found in [Roughan et al., 2002], where data have been collected on a backbone network during more than one year. This allowed the authors to quantify the traffic into a regular, predictable component and a stochastic component and to generate good traffic estimation.

In our problem, we consider that the traffic regularly evolves. Up until now we consider that traffic evolutions are sufficiently long for us to have time to implement the solutions. This approach is justified by the fact that we consider circuit-oriented communications.

2.2. Technological aspects

Consequence of such traffic evolution, the initial routing and virtual topology may not remain the optimal one. With time, the situation may get worse, leading to a loss of performance or an increase of the probability of rejecting traffic. Therefore, it becomes necessary to reconfigure the network, changing the virtual topology and the routing.

The reconfiguration problem is to find out how to change the routing and virtual topology, to keep them optimal regarding the traffic. The switches configuration is deduced, from the new virtual topology. However, it may not be desirable to modify the virtual topology completely due to its implications as it may generate network disruptions. Considering the huge quantity of data flowing constantly in a backbone, such an interruption must be as short as possible. The modifications have to be sufficient without being excessive.

We are facing a problem where we have to consider the tradeoff between the number of changes to apply in the configuration of the network and the network performance. Note that we do not consider the propagation of the changes on the network [G.N. Rouskas, 1995].

3. MILP model

3.1. Source formulation

The reconfiguration problem is an evolution of the VTDR problem. A common formulation for the VTDR problem is a flow formulation [Banerjee, 1996]. In such formulation, there is a variable making the association between each commodity and each link, indicating if the first one uses the second one. In our case, there is a high number of commodities going through the network. The number of generated variables and constraints is very high.

We tried to obtain the most concise model possible. The number of variables and constraints can be reduced by aggregating all commodities from a given node. If the cost associated with each edge does not depend on the commodity, both approaches are equivalent [Rockafellar, 1998]. This leads us to a source formulation of the reconfiguration problem. Such a formulation is used for the virtual topology design problem in [Tornatore et al., 2002]. According to the authors, such formulation allows to reduce the computer memory occupation of the problem, and to solve it with less computational efforts.

Structurally, the main difference between those formulations appears when solving flow problems with a Dantzig-Wolfe decomposition algorithm [Dantiz and Wolfe, 1960]. With an origin-destination formulation, a high number of simple problems (shortest path) are solved; with a source formulation, a lower number of more complex problems (shortest path tree) are solved [Jones et al., 1992].

3.2. Notations

We consider a network as a multi-graph $\mathcal{P} = (\mathcal{V}, \mathcal{E})$ of $|\mathcal{V}|$ nodes. Each node $n \in \mathcal{V}$ corresponds to a telecommunication center. Each edge $e \in \mathcal{E}$ corresponds to an cable between two telecommunication centers (m, n), containing $\mathcal{F}_{(m,n)}$ optical fibers from

node m to n. The topology considered is arbitrary (mesh) and not necessary symmetrical: we can have $\mathcal{F}_{(m,n)} \neq \mathcal{F}_{(n,m)}$. Each optical fiber can transport simultaneously $\mathcal{W}$ wavelengths $l_1, \ldots, l_{\mathcal{W}}$. Each one can transport a bandwidth $\mathcal{C}$, expressed in Mbps. We consider that $\mathcal{W}$ and $\mathcal{C}$ are the same on the entire network: they involve many technological parameters (range of frequency used, kind of optical fiber, and so on). We believe that few telecommunication operator would build an heterogeneous network. However, it is quite simple modify our model to consider heterogeneous lightpath capacity.

Our problem is evolving over the time. The overall time period is divided in $\mathcal{T}$ periods, and changes in the data may occur each time period $t_1, \ldots, t_{\mathcal{T}}$. We consider that each time period is long enough to solve the considered problem and implement the solution. We do not consider real-time changes in the data.

For each pair $(s, d) \in \mathcal{V}^2$ and for each time period t, a demand request $\mathcal{D}_{(s,d)}(t)$, expressed in Mbps, is defined. We define a lower bound T_m and an upper bound T_M for the values of the demand request $\mathcal{D}_{(s,d)}(t)$.

When defining a virtual topology for time period t, our objective is to define a set of lightpaths $\mathcal{L}(t)$. We denote $l_w^{(i,j)}(t)$ an elementary path on G from i to j using wavelength w of each edge supporting the lightpath. The set of nodes $\mathcal{V}$ and the set of lightpaths $\mathcal{L}(t)$ define a multi-graph $\mathcal{T}(t) = (\mathcal{V}, \mathcal{L}(t))$ corresponding to the virtual topology for time period t.

The routing problem for time period t consists of finding a flow . A flow from s to d is a set of values associated to the edges of the graph $\mathcal{T}(t)$, such that the flow conservation constraints and the capacity constraints are respected. This means that for intermediary nodes, the sum of entering values is equal to the sum of outgoing values.

3.3. Variables

We define the following variables:

- $p_{(m,n),w}^i(t)$ is the number of wavelengths w used by lightpaths having node i as origin on physical link $(m, n) \in \mathcal{E}$, for time period t. We can consider that $i \neq n$, since traffic from node i will unlikely use a lightpath having i as a destination. It generates $|\mathcal{V}||\mathcal{E}|\mathcal{W}\mathcal{T} = O\left(|\mathcal{V}|^3\mathcal{W}\mathcal{T}\right)$ variables.
- $c_w^{(i,j)}(t)$ is the number of lightpaths from node i to node j using wavelength w for time period t. It generates $(|\mathcal{V}| - 1)|\mathcal{V}|\mathcal{W}\mathcal{T} = O\left(|\mathcal{V}|^2\mathcal{W}\mathcal{T}\right)$ variables.
- $c^{(i,j)}(t)$ is the number of lightpaths from node i to node j for time period t. It generates $(|\mathcal{V}| - 1)|\mathcal{T} = O\left(|\mathcal{V}|^2\mathcal{T}\right)$ variables.
- $f_{(i,j)}^s(t)$ is the flow from source s using lightpath (i, j) for time period t. We can consider that $s \neq j$, since traffic from node s will not use lightpath having s for destination. It generates $(|\mathcal{V}| - 1)|\mathcal{V}|^2\mathcal{T} = O\left(|\mathcal{V}|^3\mathcal{T}\right)$ variables.
- $\Delta p_{(m,n),w}^i(t)$ is the number of changes for the number of wavelengths w used by lightpaths having node i as a source on physical link $(m, n) \in \mathcal{E}$, between time period $t - 1$ and t. It generates $|\mathcal{V}||\mathcal{E}|\mathcal{W}(\mathcal{T} - 1) = O\left(|\mathcal{V}|^3\mathcal{W}\mathcal{T}\right)$ variables.
- $\Delta f_{(i,j)}^s(t)$ is the demand from source s using lightpath (i, j) between time period $t - 1$ and period t. It generates $(|\mathcal{V}| - 1)|\mathcal{V}|^2\mathcal{W}(\mathcal{T} - 1) = O\left(|\mathcal{V}|^3\mathcal{T}\right)$ variables.

The overall number of generated variables is $O\left(|\mathcal{V}|^3\mathcal{W}\mathcal{T}\right)$.

3.4. Virtual topology design constraints

The constraints associated with the virtual topology design problem are the following:

$$\sum_{(i,n)\in\mathcal{E}}\sum_{w=1}^{\mathcal{W}} p^i_{(i,n),w}(t) = \sum_{j\in\mathcal{V}} c^{(i,j)}(t), \quad \begin{array}{l}\forall i \in \mathcal{V}\\ 1 \leqslant t \leqslant \mathcal{T}\end{array} \qquad (1)$$

$$\sum_{(m,n)\in\mathcal{E}} p^i_{(m,n),w}(t) - \sum_{(n,p)\in\mathcal{E}} p^i_{(n,p),w}(t) = c^{(i,n)}_w(t), \quad \begin{array}{l}\forall i,n \in \mathcal{V}^2, i \neq n\\ 1 \leqslant w \leqslant \mathcal{W}\\ 1 \leqslant t \leqslant \mathcal{T}\end{array} \qquad (2)$$

$$\sum_{w=1}^{\mathcal{W}} c^{(i,j)}_w(t) = c^{(i,j)}(t), \quad \begin{array}{l}\forall i,j \in \mathcal{V}^2, i \neq j\\ 1 \leqslant t \leqslant \mathcal{T}\end{array} \qquad (3)$$

$$\sum_{i\in\mathcal{V},i\neq n} p^i_{(m,n),w}(t) \leqslant \mathcal{F}_{(m,n)}, \quad \begin{array}{l}\forall (m,n) \in \mathcal{E}\\ 1 \leqslant w \leqslant \mathcal{W}\\ 1 \leqslant t \leqslant \mathcal{T}\end{array} \qquad (4)$$

Constraints (1) corresponds to the flow conservation for each source node i. Constraints (2) corresponds to the flow conservation in destination nodes n, for each wavelengths. Constraints (3) corresponds to the number of lightpath conservation. Constraints (4) corresponds to the capacity constraints.

As we consider multi-fiber networks, we have to consider the capacity wavelength by wavelength: We cannot allow twice the same wavelength in a given fiber, and consequently we cannot allow each wavelength more than there are fibers installed. Figure 3 illustrates this: it is not possible to allocate more wavelength l_1 between A and B, but there is still capacity available, since it is possible to allocate a wavelength l_2.
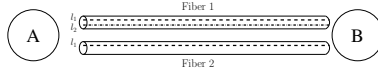

Figure 3: Capacity constraints have to be considered for each wavelength

The number of constraints generated for the virtual topology design problem is $O\left(|\mathcal{V}|^2 \mathcal{W}\mathcal{T}\right)$.

3.5. Routing constraints

$$\sum_{j\in\mathcal{V},j\neq s} f^s_{(s,j)}(t) = \sum_{d\in\mathcal{V},d\neq s} \mathcal{D}_{(s,d)}(t), \quad \forall s \in \mathcal{V}, 1 \leqslant t \leqslant \mathcal{T} \qquad (5)$$

$$\sum_{i\in\mathcal{V},i\neq s} f^s_{(i,k)}(t) - \sum_{j\in\mathcal{V},j\neq s} f^s_{(k,j)}(t) = \mathcal{D}_{(s,k)}(t), \quad \begin{array}{l}\forall (s,k) \in \mathcal{V}^2, k \neq s\\ 1 \leqslant t \leqslant \mathcal{T}\end{array} \qquad (6)$$

$$\sum_{s\in\mathcal{V},s\neq j} f^s_{(i,j)}(t) \leqslant C\sum_{w=1}^{\mathcal{W}} c^{(i,j)}_w(t), \quad \forall (i,j) \in \mathcal{V}^2, 1 \leqslant t \leqslant \mathcal{T} \qquad (7)$$

Constraints (5) corresponds to the flow conservation constraints in source node s. Constraint (6) corresponds to flow conservation in destination node k. Finally, constraints (7) is the capacity constraint.

The number of constraints generated for the routing is $O\left(|\mathcal{V}|^2 \mathcal{T}\right)$.

3.6. Reconfiguration constraints

We consider the reconfiguration problem as a succession of VTDR problems. We can add the following constraints, in order to express the lightpath changes that occur from a time-period to another.

$$p^i_{(m,n),w}(t) - p^i_{(m,n),w}(t-1) \leqslant \Delta p^i_{(m,n),w}(t), \quad \begin{matrix} \forall i \in \mathcal{V}, (m,n) \in \mathcal{E}, i \neq n \\ 1 \leqslant w \leqslant \mathcal{W} \\ 2 \leqslant t \leqslant \mathcal{T} \end{matrix} \tag{8}$$

$$p^i_{(m,n),w}(t-1) - p^i_{(m,n),w}(t) \leqslant \Delta p^i_{(m,n),w}(t), \quad \begin{matrix} \forall i \in \mathcal{V}, (m,n) \in \mathcal{E}, i \neq n \\ 1 \leqslant w \leqslant \mathcal{W} \\ 2 \leqslant t \leqslant \mathcal{T} \end{matrix} \tag{9}$$

$$f^s_{(i,j)}(t) - f^s_{(i,j)}(t-1) \leqslant \Delta f^s_{(i,j)}(t), \quad \begin{matrix} \forall (s,i,j) \in \mathcal{V}^3 \\ 2 \leqslant t \leqslant \mathcal{T} \end{matrix} \tag{10}$$

$$f^s_{(i,j)}(t-1) - f^s_{(i,j)}(t) \leqslant \Delta f^s_{(i,j)}(t), \quad \begin{matrix} \forall (s,i,j) \in \mathcal{V}^3 \\ 2 \leqslant t \leqslant \mathcal{T} \end{matrix} \tag{11}$$

The number of constraints generated for the virtual topology design is $O\left(|\mathcal{V}|^2\mathcal{W}\mathcal{T}\right)$.

3.7. Metrics

For the reconfiguration problem, various metrics can be used.

Number of wavelengths The number of used wavelengths is a commonly used metric and represents the number of transmitters and receivers needed. It has direct influence on the cost of the switches used. We can express this metric in the following way:

$$f_1(t) = \sum_{i \in \mathcal{V}} \sum_{(m,n) \in \mathcal{E}} \sum_{w=1}^{\mathcal{W}} p^i_{(m,n),w}(t), \ 1 \leqslant t \leqslant \mathcal{T} \tag{12}$$

Maximum link load in number of lightpaths Minimizing the maximum link load in number of lightpaths allows to distribute the lightpaths between all the links. That avoids having a small set of links carrying all lightpaths. Well-distributed lightpaths makes network evolution and management more flexible, since there is capacity available in all links. It allows to perform easily load balancing, to allocate dedicated protection paths, and so on. Let us call $f_2(t) = M_l(t)$ the maximum link load for time period t. We need to include the following constraint:

$$\sum_{i \in \mathcal{V}} \sum_{w=1}^{\mathcal{W}} p^i_{(m,n),w}(t) \leqslant M_l(t), \ \forall\, (m,n) \in \mathcal{E}, 1 \leqslant t \leqslant \mathcal{T} \tag{13}$$

Average number of hops The average number of hops has a direct influence on the transmission time [Banerjee, 1996, Geary et al., 2001]. In our model, we consider that a signal goes through electronic devices only when it enters or leaves a lightpath. Going through an electronic device is considered as "slow", as it requires optical-electronic conversions. The number of hops of a demand from s to d is the number of lightpaths that the signal goes through. We can express it with the following constraint.

$$f_3(t) = \frac{1}{\sum_{s \in \mathcal{V}} \sum_{d \in \mathcal{V}} \mathcal{D}_{(s,d)}(t)} \sum_{s \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} f^s_{(i,j)}(t), \ 1 \leqslant t \leqslant \mathcal{T} \tag{14}$$

Maximum lightpath load Minimizing the maximum lightpath load allows a given flexibility, by avoiding lightpaths to be completely filled. It also decreases load of switches at the extremities of lightpaths, since a given switch has less packets to route [Baldine and Rouskas, 1999a].

However we could not find a way to express it without changing the structure of our model.

Number of lightpath changes All metrics above are metrics for the VTDR problem. However, the reconfiguration problem introduces some specific metrics, such as the number of changes that have to be applied to the virtual topology, from a time period to other. We can express in the following way:

$$f_4(t) = \sum_{i \in \mathcal{V}} \sum_{(m,n) \in \mathcal{E}} \sum_{w=1}^{\mathcal{W}} \Delta p^i_{(m,n),w}(t), \ 2 \leqslant t \leqslant \mathcal{T} \tag{15}$$

Number of routing changes Another metric specific to the reconfiguration problem is the number of changes in the routing of packets. This can be expressed in the following way:

$$f_5(t) = \sum_{s \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \Delta f^s_{(i,j)}(t), \ 2 \leqslant t \leqslant \mathcal{T} \tag{16}$$

3.8. Integrality constraints

Some of the variables considered have to be integer:

- $p^i_{(m,n),w}(t) \in \mathbb{N}$
- $c^{(i,j)}_w(t) \in \mathbb{N}$
- $c^{(i,j)}(t) \in \mathbb{N}$
- $\Delta p^i_{(m,n),w}(t) \in \mathbb{N}$

With these constraints our linear programming formulation is a MILP (Mixed Integer Linear Programming) problem. While linear programming problems have a polynomial complexity, MILP problems have generally an exponential complexity.

However, we can relax this integrality constraint for some variables. $c^{(i,j)}_w(t)$ will necessary be integer since they are the sum of integer variables. Solving the problem without the associated integrality constraint, $\Delta p^i_{(m,n),w}(t)$ variables may not be integer. However, as we want to minimize the number of changes, the minimum will be reached only for an integer value.

Doing so, the number of integer variables is $O\left(|\mathcal{V}|^3 \mathcal{W}\mathcal{T}\right)$, and the number of continuous variables is $O\left(\left(|\mathcal{V}|^2 |\mathcal{W}| + |\mathcal{V}|^3\right) \mathcal{T}\right)$.

4. Additional cuts

The number of variables and constraints generated is high, and any solver will have some difficulties to solve the problem for medium sized and large instances. To improve the solving of such problems, we added cuts to our model. A cut is an additional constraint reducing the solution space without excluding the optimal solution. Adding adapted cuts to the mathematical formulation helps the solver in discovering earlier that a part of the exploration tree will not lead to valid integer solution.

4.1. Number of lightpaths required

The sum of the demands from node s is a lower bound for the overall traffic leaving s. Similarly, the sum of the demands to node d is a lower bound for the overall traffic arriving in d. As a lightpath as a fixed capacity, this imply a lower bound on the number of lightpaths from s (traffic leaving s - constraints (17)) and to d (traffic reaching d - constraints (18)).

$$\sum_{(i,n)\in\mathcal{E}}\sum_{w=1}^{\mathcal{W}} p^i_{(i,n),w}(t) \geqslant \left\lceil \frac{\sum_{d\in\mathcal{V}}\mathcal{D}_{(i,d)}(t)}{\mathcal{C}} \right\rceil, \ \forall i \in \mathcal{V}, 1 \leqslant t \leqslant \mathcal{T} \tag{17}$$

$$\sum_{i\in\mathcal{V}i\neq j} c^{(i,j)}_w(t) \geqslant \left\lceil \frac{\sum_{s\in\mathcal{V}}\mathcal{D}_{(i,d)}(t)}{\mathcal{C}} \right\rceil, \ \forall j \in \mathcal{V}, 1 \leqslant t \leqslant \mathcal{T} \tag{18}$$

4.2. Flow and number of lightpaths

An equation relating the flow variables and the number of lightpaths can be defined. It "helps" making the $c^{(i,j)}_w(t)$ being equal to zero if the flow variable is equal to zero. This cut can be expressed this way:

$$f^s_{(i,j)}(t) \leqslant \sum_{d\in\mathcal{V}}\mathcal{D}_{(s,d)}(t)\sum_{w=1}^{\mathcal{W}} c^{(i,j)}_w(t), \ \forall(s,i,j) \in \mathcal{V}^3, s \neq j, 1 \leqslant t \leqslant \mathcal{T} \tag{19}$$

4.3. A lower bound

The number of variables and restrictions of our model grows linearly with the number of wavelengths a fiber is able to transmit. Instead of having $\mathcal{F}_{(m,n)}$ fibers of capacity $\mathcal{W}$ from nodes m to node n, we could consider that there are $\mathcal{W}\mathcal{F}_{(m,n)}$ fibers installed, each one able to transmit one wavelength. The traffic considered remains the same. The overall capacity of the network remains the same, but it is impossible to have two conflicting wavelengths in the same fiber. Solving such problem is easier than solving the original problem. The solution obtained may not be feasible for the original problem, but gives a lower bound for our problem [Jaumard et al., 2004].

Having such lower bound helps use to have an estimation of the performance of simple heuristics. It may also improve the lower bound computed by the solver.

5. Experiments

We made some experiments with the networks represented in figures 4 and 5. We did not succeed in achieving significant results with bigger networks. We believe that it is possible to solve the problem for networks having up to 20 nodes and 3 or 4 time periods.

For both networks, the following parameters were chosen:
- $\mathcal{F}_{(m,n)} = 4, \forall(m,n) \in \mathcal{V}^2$;
- $\mathcal{W} = 32$ for the small network and $\mathcal{W} = 16$ for the medium network;
- $\mathcal{C} = 40$Gbps

For each time period, the traffic between two nodes s and d has been randomly chosen between 20 and 150Gbps. Due to the very large number of variables generated, we considered only 2 time periods.

The results obtained are compared with results obtained with a simple greedy algorithm, and an adaptative algorithm adapted from [Gençata and Mukherjee, 2002].
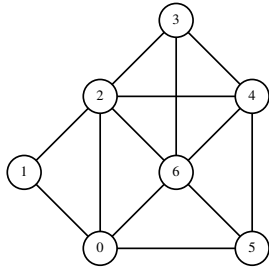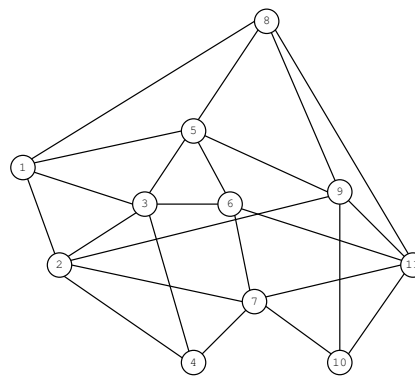
Figure 4: Small network: 7 nodes and 13 edges

Figure 5: Medium network: 11 nodes and 42 edges

5.1. Greedy algorithm

We designed a greedy algorithm. Algorithms of this class are generally simple algorithms. Their main characteristic is the fact that we do not go back in our choices. In our solving problem: we have a method allowing us to iteratively create lightpaths and then to route demands on them. But once a lightpath has been created, it will not be deleted, even if this lead to a bad solution.

Algorithm 1 gives a high-level description of our greedy algorithm for the VTDR problem. This heuristic has been designed to minimizes the number of hops. It can be easily adapted to become a reconfiguration algorithm, as the greedy algorithm is iterative, it is able to go on configuring a network from almost any initial situation. This greedy reconfiguration algorithm is described by algorithm 2.

5.2. Adaptative algorithm

This algorithm has been defined in [Gençata and Mukherjee, 2002], to perform dynamically reconfiguration without knowing the evolution of the traffic. In other words, it is an adaptive algorithm, for continuous and small changes in traffic. It tries to preserve a "reasonable" (neither too high nor too low) maximum lightpath load.

We altered the algorithm to consider the static case, in the following way: we simulate evolution from traffic matrix in time period t to time period $t + 1$ step by step with a given granularity, and run the algorithm each step.

5.3. Computational results

We run an optimization process for both networks with each of the metric. We used Cplex[1] version 7 on SunBlade Ultra-Sparc 500HMHz. We limited the computation time to 8000 seconds for each tests. The number of variables and constraints of our problems is written in table 1. The execution time for the greedy and the adaptative algorithm is almost instantaneous.

Table 1: Number of variables and constraints

	Number of variables		Number of
	Integer	Float	constraints
Small network	18816	13085	24136
Medium network	38720	26761	49284

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[1]Copyright ©Ilog 1997-2001. Cplex is a registered trademark of Ilog.

Algorithm 1 Greedy algorithm solving the VTDR problem

Require: A physical topology and a traffic matrix
Ensure: A virtual topology or a `fail`
  for all demands greater than the capacity of a lightpath **do**
    if it is possible to build a direct lightpath (s, d) respecting link capacity **then**
      Build a lightpath from s to d using shortest path
      Fill this lightpath with demand from s to d
    end if
  end for
  for all remaining demands **do** {We try to route on direct lightpath}
    if it is possible to build a direct lightpath (s, d) respecting link capacity **then**
      Build a lightpath from s to d using shortest path
      Route unrouted demand from s to d on this lightpath
    end if
  end for
  for all remaining demands **do** {We try to use direct lightpath}
    if There is capacity available on any direct lightpath from s to d **then**
      Route unrouted demand from s to d on those lightpath
    end if
  end for
  for all remaining demands **do**
    if It is possible to build a path from s to d **then**
      Build a set of lightpath from s to d, minimizing the number of hops
      Route unrouted demands with this set of lightpaths
    end if
  end for
  for all remaining demands **do**
    if There exists a path on virtual topology from s to d **then**
      Route unrouted demands with shortest path algorithm on logical topology
    end if
  end for
  if There is unrouted demands **then**
    `fail`
  end if

Algorithm 2 Greedy algorithm for reconfiguration

Require: A physical topology, a solution for the VTDR problem, a traffic evolution
Ensure: A new virtual topology and routing or `fail`
  for All demand which decreased between $t - 1$ and t **do** {We first free resources}
    Unallocate demand to reach new value for traffic. By order of preference, unallocate:
      •   multi-hop routing, by decreasing number of hop
      •   routings sharing links with other demands
      •   routings not filling completely lightpath
  end for
  Delete empty lightpaths
  for All demands **do**
    Increases routing on existing lightpaths with available capacity
    Apply Greedy algorithm the the VTDR problem
  end for

First and foremost, we observed that the computation time highly depends on the objective function chosen. It seems that minimizing the number of wavelengths used is much harder than minimizing the maximum link load or the number of changes. The time-limit has always been hit when using this metric, even for the small network. It also has been reached when minimizing the maximum link load for the medium network.

Figures 6 and 7 represent for the small network the results obtained with our model, our greedy algorithm and our adaptation of the algorithm from [Gençata and Mukherjee, 2002] for the small network.
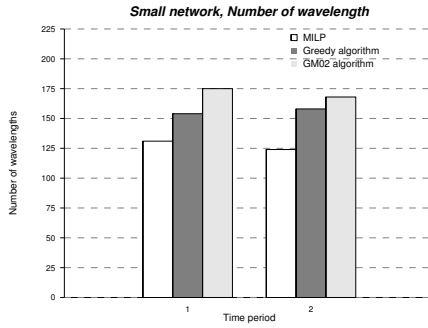
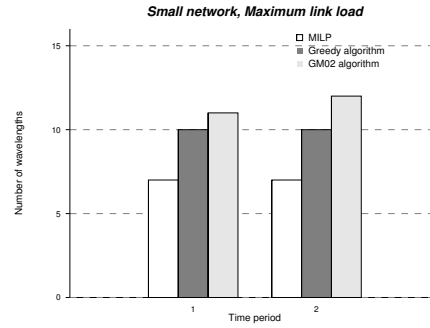**Figure 6:** **Small network. Num. of wavelengths**

Figure 7: **Small network. Maximum link load**

Our algorithm always performs significantly better than the two heuristics. However, the computation time for the heuristics is almost instantaneous. This is not the case for the exact solving. We represented on figure 8 for each metric the sum for each time period of the number of used wavelengths, and the number of lightpaths change. We can observe that when minimizing the number of lightpaths change, the number of used wavelengths is very high, and the number of changes is very low. This happens because the solver tends to use available capacity to allocate lightpaths that will be used for the other time periods. Similarly if we want to minimize the number of re-routings, the number of used wavelengths is very high. This time, it happens because the virtual topology is designed in a way allowing the reuse of the lightpaths; that is many undirect lightpaths are created. Such results illustrates the tradeoff between the performance loss and the number of changes to apply to reconfigure the network.
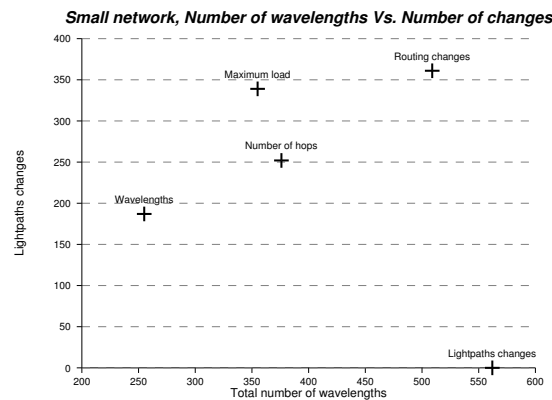

Figure 8: Small network: number of wavelengths and number of changes

The results for the medium network are similar. We represented the same data (see figures 9 and 10).
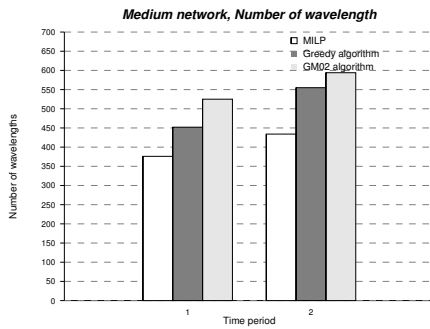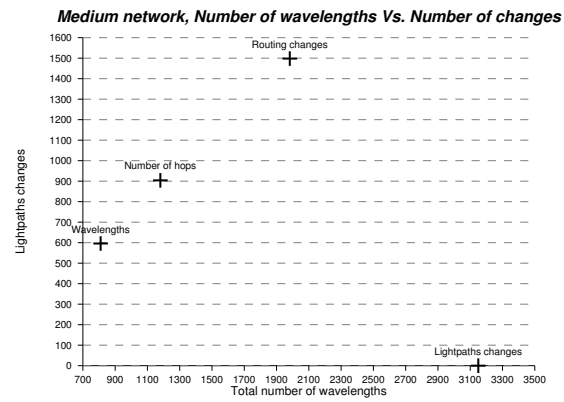
Figure 9: Medium network. Num. of wavelengths

Figure 10: Medium network: Num. of wavelengths and num. of changes

6. Conclusion

We aim to solve the reconfiguration problem for multi-fiber WDM networks. We present the problem and propose a MILP formulation solving it. We compare the performance of the exact solution with a simple greedy algorithm and with an algorithm adapted from the literature.

The problems generated by our model are large, and we have to restrict ourself to small and medium sized networks. However, the results obtained with the MILP formulation significantly outperform the ones obtained with simple heuristics. Our experiments also illustrate the tradeoff between the performance of the network and the number of changes to apply to the logical topology in order to carry a new traffic.

Our results emphasize the advantages of an exact approach for the reconfiguration problem. We intend to develop specific techniques, such as the decomposition, in order to decrease the computation time for solving the problem. We also intend to make a multiobjective study of the problem It would allow us to find out the relationship between the network performance and the number of changes to apply to the network to adapt to new traffics.

References

Baldine, I. and Rouskas, G. (1999a). Dynamic reconfiguration policies for WDM networks. In *IEEE Infocom*, pages 313–320.

Baldine, I. and Rouskas, G. (1999b). Reconfiguration and dynamic load balancing in broadcast WDM networks. *Photonic Network Communications Journal*, 1(1):49–64.

Banerjee, D. (1996). *Design and analysis of wavelength-routed optical networks*. PhD thesis, University of California, Davis.

Beauquier, B. (2000). *Communication dans les réseaux optiques par multiplexage en longueur d'onde*. PhD thesis, Université de Nice Sophia Antipolis.

Chlamtac, I., Ganz, A., and G, K. (1992). Lightpath communications: An approach to high-bandwidth optical WAN's. *lEEE Transactions on Communications*, 40(7):1171–1182.

Dantiz, G. and Wolfe, P. (1960). Decomposition principle for linear programs. *Operation Research*, 8:101–111.

Dutta, R. and Rouskas, G. (1999). A survey of virtual topology design algorithms for wavelength routed optical networks. Technical Report TR-99-06, NCSU CSC.

Geary, N., Antonopoulos, A., Drakopoulos, E., and O'Reilly, J. (2001). Analysis of optimization issues in multi-period DWDM network planning. In *IEEE Infocom*, pages 152–158. IEEE.

Gençata, A. and Mukherjee, B. (2002). Virtual-topology adaptation for WDM mesh networks under dynamic traffic. In *IEEE Infocom*.

G.N. Rouskas, M. A. (1995). Dynamic reconfiguration in multihop wdm networks. *Journal of High Speed Networks*, 4(3):221–238.

Jaumard, B., Meyer, C., and Thiongane, B. (2004). ILP formulations for the RWA problem-symmetyric systems. In *Globecom 2004*. IEEE.

Jones, K., Lustig, I., Farvolden, J., and Powell, W. (1992). Multicommodity networks flows: The impact of formulation on decomposition. Technical Report SOR-91-23, Department of Civil Engineering and Operations Research, Princeton University.

Li, G. and Simha, R. (2001). On the wavelength assignment problem in multifiber wdm star and ring networks. *IEEE/ACM Transactions on Networking*, 9(1):60–68.

Narula-Tam, A. and Modiano, E. (2000). Dynamic load balancing in WDM packet networks with and without wavelength constraints. *IEEE Journal on selected areas in communications*, 18(10):1972–1979.

Ramaswami, R. and Sivarajan, K. (1998). *Optical networks. A practical perspective*. Morgan Kaufmann Publishers, academic press edition.

Rockafellar, R. (1998). *Network flows and monotropic optimization*. Athena scientific.

Roughan, M., Greenberg, A., Kalmanek, C., Rumsewicz, M., Yates, J., and Zhang, Y. (2002). Experience in measuring backbone traffic variability: models, metrics, measurements and meaning. In *ACM SIGCOMM Internet Measurement Workshop (IMW'2002)*. (extended abstract).

Tornatore, M., Maier, G., and Pattavina, A. (2002). WDM network optimization by ILP based on source formulation. In *Infocom*. IEEE.

Télécom, F. (2002a). Document de référence 2001. Technical report, France Télécom.

Télécom, F., editor (2002b). *Les communications optiques du futur*. Mémento technique France Télécom 19. http://www.rd.francetelecom.fr/fr/conseil/mento19/index.html.