6

Natural and Step Responses of RLC Circuits

Drill Exercises

DE 6.1 **[a]** $\dfrac{1}{(2RC)^2} = \dfrac{1}{LC}, \qquad \text{therefore} \quad C = 500\,\text{nF}$

[b] $\alpha = 5000 = \dfrac{1}{2RC}, \qquad \text{therefore} \quad C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\dfrac{1}{\sqrt{LC}} = 20{,}000, \qquad \text{therefore} \quad C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

DE 6.2

$$\begin{aligned} i_\text{L} &= \frac{1000}{50}\int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\ &= 20\left\{\frac{-14e^{-5000x}}{-5000}\bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\bigg|_0^t\right\} + 30 \times 10^{-3} \\ &= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\ &= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\ &= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0 \end{aligned}$$

DE 6.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40$ krad/s.

[a] $v(0^-) = v(0^+) = 0, \qquad \text{therefore} \quad i_\text{R}(0^+) = 0$

[b] $i_C(0^+) = 4\,\text{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = 4,\qquad \text{therefore}\quad \dfrac{dv_c(0^+)}{dt} = 4\times 10^8\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V},\qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2,\qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $A_1 + A_2 = 0,\quad -A_1 - 4A_2 = 40{,}000,\quad A_1 = 40{,}000/3$

[e] $A_2 = -40{,}000/3$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V},\qquad t \geq 0^+$

DE 6.4 **[a]** $\dfrac{1}{2RC} = 8000,\qquad \text{therefore}\quad R = 62.5\,\Omega$

[b] $i_R(0^+) = \dfrac{10}{62.5} = 160\,\text{mA}$

$$i_C(0^+) = -80 - 160 = -240\,\text{mA},\qquad i_C(0^+) = C\frac{dv(0^+)}{dt}$$

Therefore $\dfrac{dv(0^+)}{dt} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V},\qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000,\qquad B_2 = (-80/3)\,\text{V}$

[d] $i_L = -(i_R + i_C);\qquad i_R = v/R;\qquad i_C = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

Therefore $i_R = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\text{mA}$

$$i_C = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_L = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA},\qquad t \geq 0$$

DE 6.5 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4},\qquad \text{therefore}\quad \dfrac{1}{2RC} = 500,\quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5\times 10^{-3},\qquad \text{therefore}\quad V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5\times 10^{-3},\qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \frac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$i_R(0^+) = \frac{50}{100} = 500\,\text{mA}$

Therefore $\quad i_C(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\quad \frac{dv(0^+)}{dt} = -750 \times \frac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $\quad D_1 - \alpha D_2 = -75{,}000;$

$\alpha = \frac{1}{2RC} = 500, \quad D_1 = -50{,}000\ \text{V/s}$

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$i_R = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$

DE 6.6 **[a]** $i_R(0^+) = \frac{V_0}{R} = \frac{40}{500} = 0.08\,\text{A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\frac{di_L(0^+)}{dt} = \frac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \frac{1}{2RC} = 1000; \qquad \frac{1}{LC} = 1{,}562{,}500;$

$s_{1,2} = -1000 \pm j750\ \text{rad/s}$

[e] $i_L = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t, \qquad i_f = -1\,\text{A}$

$i_L(0^+) = 0.5 = i_f + B_1', \qquad \text{therefore} \quad B_1' = 1.5\,\text{A}$

$\frac{di_L(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad \text{therefore} \quad B_2' = (25/12)\,\text{A}$

$\therefore \quad i_L(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0^+$

[f] $v(t) = \frac{L di_L}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t] V \qquad t \geq 0$

DE 6.7 **[a]** $i(0^+) = 0$

[b] $v_c(0^+) = v_C(0^-) = \left(\frac{80}{24}\right)(15) = 50\,\text{V}$

[c] $50 + L\frac{di(0^+)}{dt} = 100, \qquad \frac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \frac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000\ \text{rad/s}$

[e] $i = i_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $B_1' = 0; \qquad \dfrac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\text{A}, \quad t \geq 0$

DE 6.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t], \qquad v_f = 100\,\text{V}$

$v_c(0^+) = 50\,\text{V}; \qquad \dfrac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$

$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$

Therefore $B_2' = \dfrac{\alpha}{\omega_d}B_1' = \left(\dfrac{8000}{6000}\right)(-50) = -66.67\,\text{V}$

Therefore $v_c(t) = 100 - e^{-8000t}[50\cos 6000t + 66.67\sin 6000t]\,\text{V}, \quad t \geq 0$

Problems

P 6.1 **[a]** $\alpha = \dfrac{1}{2RC} = 1 \text{ rad/s}$

$\omega_o^2 = \dfrac{1}{LC} = 10$

$\omega_d = \sqrt{10 - 1} = 3 \text{ rad/s}$

$\therefore \quad v = B_1 e^{-t}\cos 3t + B_2 e^{-t}\sin t$

$v(0) = B_1 = 0; \qquad v = B_2 e^{-t}\sin 3t$

$i_R(0^+) = 0\,\text{A}; \qquad i_C(0^+) = 3\,\text{A}; \qquad \dfrac{dv}{dt}(0^+) = \dfrac{3}{0.25} = 12 \text{ V/s}$

$12 = -\alpha B_1 + \omega_d B_2 = -1(0) + 3B_2$

$\therefore \quad B_2 = 4$

$\therefore \quad v = 4e^{-t}\sin 3t\,\text{V}, \qquad t \geq 0$

[**b**] $\dfrac{dv}{dt} = 4e^{-t}(3\cos 3t - \sin 3t)$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 3\cos 3t = \sin 3t \quad \text{or} \quad \tan 3t = 3$$

$$\therefore \quad 3t_1 = 1.25, \qquad t_1 = 416.35\,\text{ms}$$

$$3t_2 = 1.25 + \pi, \qquad t_2 = 1463.55\,\text{ms}$$

$$3t_3 = 1.25 + 2\pi, \qquad t_3 = 2510.74\,\text{ms}$$

[**c**] $t_3 - t_1 = 2094.40\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{3} = 2094.40\,\text{ms}$

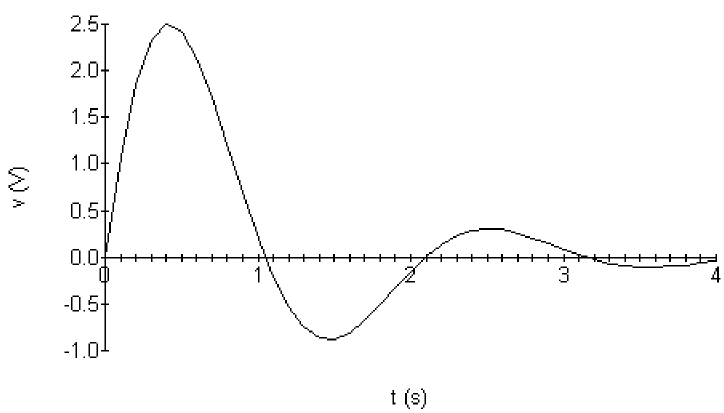
[**d**] $t_2 - t_1 = 1047.20\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{2094.40}{2} = 1047.20\,\text{ms}$

[**e**] $v(t_1) = 4e^{-(0.41635)}\sin 3(0.41635) = 2.50\,\text{V}$

$$v(t_2) = 4e^{-(1.46355)}\sin 3(1.46355) = -0.88\,\text{V}$$

$$v(t_3) = 4e^{-(2.51074)}\sin 3(2.51074) = 0.31\,\text{V}$$

[**f**]



P 6.2 [**a**] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10} = 3.16\,\text{rad/s}$

$$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \qquad v(0) = B_1 = 0; \qquad v = B_2\sin\omega_o t$$

$$C\frac{dv}{dt}(0) = -i_{\text{L}}(0) = 3$$

$$12 = -\alpha B_1 + \omega_d B_2 = -0 + \sqrt{10}B_2$$

$$\therefore \quad B_2 = 12/\sqrt{10} = 3.79\,\text{V}$$

$$v = 3.79\sin 3.16t\,\text{V}, \qquad t \geq 0$$

[b] $2\pi f = 3.16; \qquad f = \dfrac{3.16}{2\pi} \cong 0.50\,\text{Hz}$

[c] $3.79\,\text{V}$

P 6.3 **[a]** $\alpha = 4000; \qquad \omega_d = 3000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^6 + 16 \times 10^6 = 25 \times 10^6$$

$$\frac{1}{LC} = 25 \times 10^6$$

$$L = \frac{1}{(25 \times 10^6)(50 \times 10^{-9})} = 0.8\,\text{H} = 800\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{10^9}{(8000)(50)} = 2500\,\Omega$$

[c] $V_o = v(0) = 125\,\text{V}$

[d] $I_o = i_\text{L}(0) = -i_\text{R}(0) - i_\text{C}(0)$

$$i_\text{R}(0) = \frac{V_o}{R} = \frac{125}{2.5} \times 10^{-3} = 50\,\text{mA}$$

$$i_\text{C}(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 125\{e^{-4000t}[-3000\sin 3000t - 6000\cos 3000t] -$$

$$4000e^{-4000t}[\cos 3000t - 2\sin 3000t]$$

$$\frac{dv}{dt}(0) = 125\{1(-6000) - 4000\} = -125 \times 10^4$$

$$C\frac{dv}{dt}(0) = -125 \times 10^4 (40 \times 10^{-9}) = -6250 \times 10^{-5} = -62.5\,\text{mA}$$

$$\therefore \quad I_o = -50 + 62.5 = 12.5\,\text{mA}$$

[e]
$$\frac{dv}{dt} = 125e^{-4000t}[5000\sin 3000t - 10{,}000\cos 3000t]$$

$$= 625 \times 10^3 e^{-4000t}[\sin 3000t - 2\cos 3000t]$$

$$C\frac{dv}{dt} = 31{,}250 \times 10^{-6} e^{-4000t}(\sin 3000t - 2\cos 3000t)$$

$$i_C(t) = 31.25e^{-4000t}(\sin 3000t - 2\cos 3000t)\,\text{mA}$$

$$i_R(t) = 50e^{-4000t}(\cos 3000t - 2\sin 3000t)\,\text{mA}$$

$$i_L(t) = -i_R(t) - i_C(t)$$

$$= e^{-4000t}(12.5\cos 3000t + 68.75\sin 3000t)\,\text{mA}, \quad t \geq 0$$

CHECK:

$$\begin{aligned} \frac{di_L}{dt} &= \{-4000e^{-4000t}[12.5\cos 3000t + 68.75\sin 3000t] \\ &\quad +e^{-4000t}[-37.5\times 10^3\sin 3000t \\ &\quad +206.25\times 10^3\cos 3000t]\times 10^{-3} \\ &= e^{-4000t}[156.25\cos 3000t - 312.5\sin 3000t] \\ L\frac{di_L}{dt} &= e^{-4000t}[125\cos 3000t - 250\sin 3000t] \\ &= 125e^{-4000t}[\cos 3000t - 2\sin 3000t]\,\text{V} \end{aligned}$$

P 6.4 **[a]** $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (4000)^2$

$$\therefore \quad C = \frac{1}{(16\times 10^6)(5)} = 12.5\,\text{nF}$$

$$\frac{1}{2RC} = 4000$$

$$\therefore \quad R = \frac{10^9}{(8000)(12.5)} = 10\,\text{k}\Omega$$

$$v(0) = D_2 = 25\,\text{V}$$

$$i_R(0) = \frac{25}{10} = 2.5\text{mA}$$

$$i_C(0) = -2.5 - 5 = -7.5\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 4000D_2 = \frac{-7.5\times 10^{-3}}{12.5\times 10^{-9}} = -6\times 10^5$$

$$\therefore \quad D_1 = -6\times 10^5 + 4000(25) = -5\times 10^5\ \text{V/s}$$

[b] $v = -5\times 10^5 te^{-4000t} + 25e^{-4000t}$

$$\frac{dv}{dt} = [20\times 10^8 t - 6\times 10^5]e^{-4000t}$$

$$i_C = C\frac{dv}{dt} = 12.5\times 10^{-9}[20\times 10^8 t - 6\times 10^5]e^{-4000t}$$

$$= (25{,}000t - 7.5)e^{-4000t}\,\text{mA}, \qquad t > 0$$

P 6.5 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -5000$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -20{,}000$$

$$\therefore \quad -2\alpha = -25{,}000$$

$$\alpha = 12{,}500\,\text{rad/s}$$

$$\frac{1}{2RC} = \frac{10^6}{2R(0.05)} = 12{,}500$$

$$R = 800\,\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 15{,}000$$

$$4(\alpha^2 - \omega_o^2) = 225 \times 10^6$$

$$\therefore \quad \omega_o = 10{,}000\,\text{rad/s}$$

$$\omega_o^2 = 10^8 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{10^8 C} = 200\,\text{mH}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -6.25e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 1.25e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 5e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

P 6.6 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{40} = 25 \times 10^6$

$$\omega_o = 5000 \text{ rad/s}$$

$$\frac{1}{2RC} = 5000; \qquad R = \frac{1}{10{,}000C}$$

$$R = \frac{10^9}{8 \times 10^4} = 12.5\,\text{k}\Omega$$

[b] $v(t) = D_1te^{-5000t} + D_2e^{-5000t}$

$$v(0) = -25\,\text{V} = D_2$$

$$\frac{dv}{dt} = (D_1t - 25)(-5000e^{-5000t}) + D_1e^{-5000t}$$

$$\frac{dv}{dt}(0) = 125 \times 10^3 + D_1 = \frac{i_\text{C}(0)}{C}$$

$$i_C(0) = -i_R(0) - i_L(0)$$

$$i_R(0) = \frac{-25}{12.5} = -2\,\text{mA}$$

$$\therefore \quad i_C(0) = 2 - (-1) = 3\,\text{mA}$$

$$\therefore \quad \frac{dv}{dt}(0) = \frac{3 \times 10^{-3}}{8 \times 10^{-9}} = 0.375 \times 10^6 = 3.75 \times 10^5$$

$$\therefore \quad 1.25 \times 10^5 + D_1 = 3.75 \times 10^5$$

$$D_1 = 2.5 \times 10^5 = 25 \times 10^4 \text{ V/s}$$

$$\therefore \quad v(t) = (25 \times 10^4 t - 25)e^{-5000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_C(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$$\frac{dv}{dt} = (25 \times 10^4 t - 25)(-5000)e^{-5000t} + e^{-5000t}(25 \times 10^4)$$

$$= (375{,}000 - 125 \times 10^7 t)e^{-5000t}$$

$$\frac{dv}{dt} = 0 \text{ when } 125 \times 10^7 t_1 = 375{,}000; \qquad \therefore \quad t_1 = 300\,\mu\text{s}$$

$$v(300\mu\text{s}) = 50e^{-1.5} = 11.16\,\text{V}$$

[d] $i_L(300\mu\text{s}) = -i_R(300\mu\text{s}) = \dfrac{11.16}{12.5} = 0.89\,\text{mA}$

$$\omega_C(300\mu\text{s}) = 4 \times 10^{-9}(11.16)^2 = 497.87\,\text{nJ}$$

$$\omega_L(300\mu\text{s}) = (2.5)(0.89)^2 \times 10^{-6} = 1991.48\,\text{nJ}$$

$$\omega(300\mu\text{s}) = \omega_C + \omega_L = 2489.35\,\text{nJ}$$

$$\omega(0) = 4 \times 10^{-9}(625) + 2.5(10^{-6}) = 5000\,\text{nJ}$$

$$\%\text{ remaining } = \frac{2489.35}{5000}(100) = 49.79\%$$

P 6.7 **[a]** $i_R(0) = \dfrac{90}{2000} = 45\text{mA}$

$$i_L(0) = -30\text{mA}$$

$$i_C(0) = -i_L(0) - i_R(0) = 30 - 45 = -15\,\text{mA}$$

[b] $\alpha = \frac{1}{2RC} = \frac{10^9}{(4000)(10)} = 25{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^9)}{(250)(10)} = 4 \times 10^8$$

$$s_{1,2} = -25{,}000 \pm \sqrt{6.25 \times 10^8 - 10^8(4)} = -25{,}000 \pm 15{,}000$$

$$s_1 = -10{,}000 \text{ rad/s}; \qquad s_2 = -40{,}000 \text{ rad/s}$$

$$v = A_1 e^{-10{,}000t} + A_2 e^{-40{,}000t}$$

$$v(0) = A_1 + A_2 = 90$$

$$\frac{dv}{dt}(0) = -10^4 A_1 - 4A_2 \times 10^4 = \frac{-15 \times 10^{-3}}{10 \times 10^{-9}} = -1.5 \times 10^6 \text{ V/s}$$

$$-A_1 - 4A_2 = -150$$

$$\therefore \quad -3A_2 = -60; \qquad A_2 = 20; \qquad A_1 = 70$$

$$v = 70e^{-10{,}000t} + 20e^{-40{,}000t} \text{ V}, \qquad t \geq 0$$

[c]

$$i_C = C\frac{dv}{dt}$$

$$= 10 \times 10^{-9}[-70 \times 10^4 e^{-10{,}000t} - 80 \times 10^4 e^{-40{,}000t}]$$

$$= -7e^{-10{,}000t} - 8e^{-40{,}000t} \text{ mA}$$

$$i_R = 35e^{-10{,}000t} + 10e^{-40{,}000t} \text{ mA}$$

$$i_L = -i_C - i_R = -28e^{-10{,}000t} - 2e^{-40{,}000t} \text{ mA}, \quad t \geq 0$$

P 6.8 $\frac{1}{2RC} = \frac{3 \times 10^9}{(25{,}000)(10)} = 12{,}000$

$$\frac{1}{LC} = 4 \times 10^8$$

$$s_{1,2} = -12{,}000 \pm j16{,}000 \text{ rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1 e^{-12{,}000t} \cos 16{,}000t + B_2 e^{-12{,}000t} \sin 16{,}000t$$

$$v(0^+) = 90 \text{ V} = B_1; \qquad i_R(0^+) = \frac{90}{(12{,}500/3)} = 21.6 \text{ mA}$$

$$i_{\rm C}(0^+) = [-i_{\rm L}(0^+) + i_{\rm R}(0^+)] = -[-30 + 21.6] = 8.4\,\text{mA}$$

$$\frac{dv(0^+)}{dt} = \frac{8.4\times10^{-3}}{10\times10^{-9}} = 840{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -12{,}000B_1 + 16{,}000B_2 = 840{,}000$$

or $-3B_1 + 4B_2 = 210; \qquad \therefore \quad B_2 = 120\,\text{V}$

$$v(t) = 90e^{-12{,}000t}\cos 16{,}000t + 120e^{-12{,}000t}\sin 16{,}000t\,\text{V}, \qquad t \ge 0$$

P 6.9 $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(5000)(10)} = 2\times10^4$

$$\alpha^2 = 4\times10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$$

Critical damping:

$$v = D_1te^{-\alpha t} + D_2e^{-\alpha t}$$

$$i_R(0^+) = \frac{90}{2500} = 36\,\text{mA}$$

$$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-30 + 36] = -6\,\text{mA}$$

$$v(0) = D_2 = 90$$

$$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2e^{-\alpha t}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_{\rm C}(0)}{C} = \frac{-6\times10^{-3}}{10\times10^{-9}} = -6\times10^5$$

$$D_1 = \alpha D_2 - 6\times10^5 = (2\times10^4)(90) - 6\times10^5 = 120\times10^4$$

$$v = (120\times10^4t + 90)e^{-20{,}000t}\,\text{V}, \qquad t \ge 0$$

P 6.10 **[a]** $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(10{,}000)(8)} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1.25)(8)} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(1.5625-1)10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000 \text{ rad/s}$$

$$s_2 = -20{,}000 \text{ rad/s}$$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 10^8 - 36 \times 10^6 = 0.64 \times 10^8$$

$$\alpha = 0.8 \times 10^4 = 8000$$

$$\frac{1}{2RC} = 8000; \qquad \therefore \quad R = \frac{10^9}{(16{,}000)(8)} = 7812.5\,\Omega$$

[d] $s_1 = -8000 + j6000$ rad/s; $\qquad s_2 = -8000 - j6000$ rad/s

[e] $\alpha = 10^4 = \frac{1}{2RC}; \qquad \therefore \quad R = \frac{1}{2C(10^4)} = 6250\,\Omega$

P 6.11 $\alpha = 2000/2 = 1000$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(2000)(18)} = 27.78\,\Omega$$

$$v(0^+) = -24\,\text{V}$$

$$i_{\text{R}}(0^+) = \frac{-24}{27.78} = -864\,\text{mA}$$

$$\frac{dv}{dt} = 2400e^{-200t} + 21{,}600e^{-1800t}$$

$$\frac{dv(0^+)}{dt} = 2400 + 21{,}600 = 24{,}000\,\text{V/s}$$

$$i_{\text{C}}(0^+) = 18 \times 10^{-6}(24{,}000) = 432\,\text{mA}$$

$$i_{\text{L}}(0^+) = -[i_{\text{R}}(0^+) + i_{\text{C}}(0^+)] = -[-864 + 432] = 432\,\text{mA}$$

P 6.12 **[a]** $2\alpha = 200; \qquad \alpha = 100\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 120; \qquad \omega_o = 80\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{200(200)} = 25\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(80)^2(25)} = 6.25\,\text{H}$$

$$i_{\text{C}}(0^+) = A_1 + A_2 = 15\,\text{mA}$$

$$\frac{di_C}{dt} + \frac{di_L}{dt} + \frac{di_R}{dt} = 0$$

$$\frac{di_C(0)}{dt} = -\frac{di_L(0)}{dt} - \frac{di_R(0)}{dt}$$

$$\frac{di_L(0)}{dt} = \frac{0}{6.25} = 0\,\text{A/s}$$

$$\frac{di_R(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_C(0)}{C} = \frac{15 \times 10^{-3}}{(200)(25 \times 10^{-6})} = 3\,\text{A/s}$$

$$\therefore \quad \frac{di_C(0)}{dt} = -3\,\text{A/s}$$

$$\therefore \quad 40A_1 + 160A_2 = 3$$

$$A_1 + 4A_2 = 75 \times 10^{-3}; \qquad \therefore \quad A_1 = -5\,\text{mA}; \qquad A_2 = 20\,\text{mA}$$

$$\therefore \quad i_C = 20e^{-160t} - 5e^{-40t}\,\text{mA}, \qquad t \geq 0$$

[**b**] By hypothesis

$$v = A_3e^{-160t} + A_4e^{-40t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{15 \times 10^{-3}}{25 \times 10^{-6}} = 600\,\text{V/s}$$

$$-160A_3 - 40A_4 = 600; \qquad \therefore \quad A_3 = -5\,\text{V}; \qquad A_4 = 5\,\text{V}$$

$$v = -5e^{-160t} + 5e^{-40t}\,\text{V}, \qquad t \geq 0$$

[**c**] $i_R(t) = \dfrac{v}{200} = -25e^{-160t} + 25e^{-40t}\,\text{mA}, \qquad t \geq 0^+$

[**d**] $i_L = -i_R - i_C$

$$i_L = 5e^{-160t} - 20e^{-40t}\,\text{mA}, \qquad t \geq 0$$

P 6.13 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$

The characteristic determinate is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinates are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $$N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$$

It follows that $$A_1 = \frac{N_1}{\Delta} = \frac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$$

and $$A_2 = \frac{N_2}{\Delta} = \frac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$$

We see from these expressions that $A_1 = A_2^*$

P 6.14 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 6.13 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (6.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 6.13 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

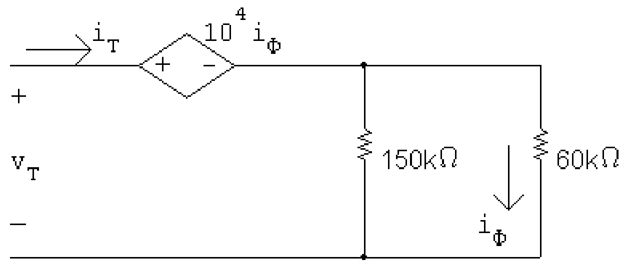
$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1$$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (6.31).

P 6.15



$$v_T = 10^4 \frac{i_T(150 \times 10^3)}{210 \times 10^3} + \frac{(150)(60)10^6}{210 \times 10^3} i_T$$

$$\frac{v_T}{i_T} = \frac{1500 \times 10^3}{210} + \frac{9000 \times 10^3}{210} = \frac{10{,}500}{210} \times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_\text{C}(0)}{C} = \frac{-0.9}{1.25} \times 10^6 = -720 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(8)(1.25)} = 10^8; \qquad \omega_o = 10^4 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(1.25) \times 10^3} = 8000 \text{ rad/s}$$

$$\omega_d = \sqrt{(100 - 64) \times 10^6} = 6000 \text{ rad/s}$$

$$v_o = B_1 e^{-8000t} \cos 6000t + B_2 e^{-8000t} \sin 6000t$$

$$v_o(0) = B_1 = 45\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 6000 B_2 - 8000 B_1 = -720 \times 10^3$$

$$\therefore \quad 6000 B_2 = 8000(45) - 720 \times 10^3; \qquad \therefore \quad B_2 = -60\,\text{V}$$

$$v_o = 45 e^{-8000t} \cos 6000t - 60 e^{-8000t} \sin 6000t\,\text{V}, \qquad t \ge 0$$

P 6.16 **[a]** $\alpha = \frac{1}{2RC} = 1250, \qquad \omega_o = 10^3, \qquad$ therefore overdamped

$s_1 = -500, \qquad s_2 = -2000$

therefore $\quad v = A_1 e^{-500t} + A_2 e^{-2000t}$

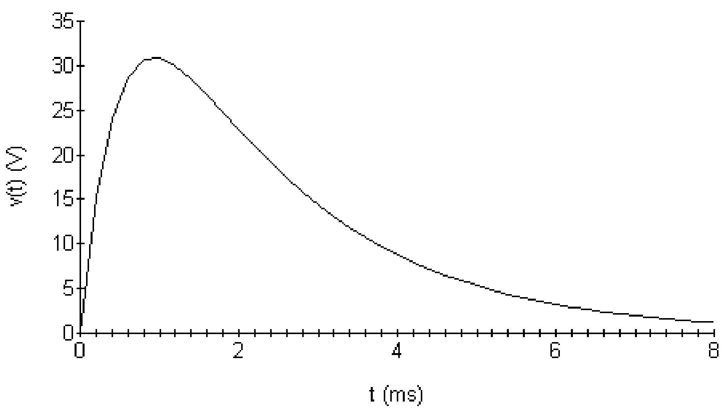
$v(0^+) = 0 = A_1 + A_2; \qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_C(0^+)}{C} = 98{,}000\,\text{V/s}$

Therefore $\quad -500A_1 - 2000A_2 = 98{,}000$

$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$

$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\,\text{V}, \qquad t \geq 0$

[b]



Example 6.4: $\quad v_{\max} \cong 74\,\text{V} \quad$ at 1.4 ms

Example 6.5: $\quad v_{\max} \cong 36.1\,\text{V} \quad$ at 1.0 ms

Problem 6.16: $\quad v_{\max} \cong 30.9 \quad$ at 0.92 ms

P 6.17 **[a]** $v = L\left(\frac{di_L}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_R = \frac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_C = I - i_L - i_R = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

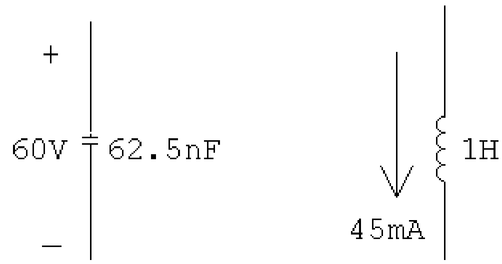
P 6.18 **[a]** $v = L\left(\frac{di_L}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,\text{V}, \qquad t \geq 0$

[b] $i_C(t) = I - i_R - i_L = 24 \times 10^{-3} - \frac{v}{625} - i_L$

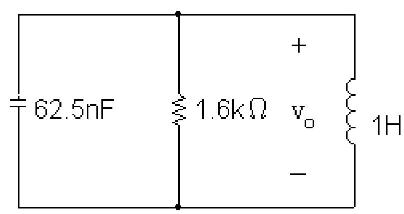
$$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$$

P 6.19 $v = L\left(\frac{di_{\rm L}}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0$

P 6.20 $t < 0: \qquad V_o = 60\,\text{V}, \qquad I_o = 45\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{60}{1600} = 37.5\,\text{mA}; \qquad i_{\rm L}(0) = 45\,\text{mA}$$

$$i_{\rm C}(0) = -37.5 - 45 = -82.5\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{3200(62.5)} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{62.5} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1e^{-2000t} + A_2e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 60$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-82.5 \times 10^{-3}}{62.5 \times 10^{-9}} = -1320 \times 10^3$$

Solving, $\qquad A_1 = -140\,\text{V}, \qquad A_2 = 200\,\text{V}$

$$\therefore \quad v_o = -140e^{-2000t} + 200e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 6.21 $$\omega_o^2 = \frac{1}{LC} = \frac{16 \times 10^6}{0.64} = 25 \times 10^6$$

$$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{4000} = 4000 \text{ nepers}; \qquad \alpha^2 = 16 \times 10^3$$

$$\omega_d = \sqrt{(25 - 16) \times 10^6} = 3000 \text{ rad/s}$$

$$s_{1,2} = -4000 \pm j3000\text{rad/s}$$

$$v_o(t) = B_1 e^{-4000t} \cos 3000t + B_2 e^{-4000t} \sin 3000t$$

$$v_o(0) = B_1 = 60\,\text{V}$$

$$i_R(0) = \frac{60}{2000} = 30\,\text{mA}$$

$$i_{\text{L}}(0) = 45\,\text{mA}$$

$$i_{\text{C}}(0) = -i_R(0) - i_{\text{L}}(0) = -75\,\text{mA}$$

$$\frac{i_{\text{C}}(0)}{C} = (-75 \times 10^{-3})(16 \times 10^6) = -12 \times 10^5$$

$$\frac{dv_o}{dt}(0) = -4000B_1 + 3000B_2 = -12 \times 10^5$$

$$\therefore \quad 3B_2 = 4B_1 - 1200 = 240 - 1200 = -960; \quad \therefore \quad B_2 = -320\,\text{V}$$

$$v_o(t) = 60e^{-4000t} \cos 3000t - 320e^{-4000t} \sin 3000t\,\text{V}, \qquad t \geq 0$$

P 6.22 $$\omega_o^2 = \frac{1}{LC} = \frac{16 \times 10^6}{0.16} = 10^8; \qquad \omega_o = 10^4$$

$$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{1600} = 10^4$$

$$\therefore \quad \alpha^2 = \omega_o^2 \text{ (critical damping)}$$

$$v_o(t) = D_1 t e^{-10{,}000t} + D_2 e^{-10{,}000t}$$

$$v_o(0) = D_2 = 60\,\text{V}$$

$$i_R(0) = \frac{60}{800} = 75\,\text{mA}$$

$$i_{\text{L}}(0) = 45\,\text{mA}$$

$$i_{\text{C}}(0) = -120\,\text{mA}$$

$$\frac{dv_o}{dt}(0) = -10{,}000D_2 + D_1$$

$$\frac{i_{\text{C}}(0)}{C} = (-120 \times 10^{-3})(16 \times 10^6) = -1920 \times 10^3$$

$$D_1 - 10{,}000D_2 = -1920 \times 10^3; \qquad D_1 = -1320 \times 10^3\text{V/s}$$

$$v_o(t) = (60 - 132 \times 10^4 t)e^{-10{,}000t}\,\text{V}, \qquad t > 0$$

P 6.23 **[a]** $2\alpha = 5000; \quad \alpha = 2500\,\text{rad/s}$

$$\sqrt{\alpha^2 - \omega_o^2} = 1500; \qquad \omega_o^2 = 4 \times 10^6; \qquad \omega_o = 2000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2500; \qquad R = 5000L$$

$$\omega_o^2 = \frac{1}{LC} = 4 \times 10^6; \qquad L = \frac{10^9}{4 \times 10^6(50)} = 5\text{H}$$

$$R = 25{,}000\,\Omega$$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(50) \times 10^{-9}v_c^2(0) = 90 \times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 3600; \qquad v_c(0) = 60\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{60}{5} = 12\,\text{A/s}$$

[c] $i(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -1000A_1 - 4000A_2 = 12$$

$$\therefore \quad A_1 + 4A_2 = -12 \times 10^{-3}$$

$$\therefore \quad A_2 = -4\,\text{mA}; \qquad A_1 = +4\,\text{mA}$$

$$i(t) = +4e^{-1000t} - 4e^{-4000t}\,\text{mA} \qquad t \geq 0$$

[d] $\dfrac{di(t)}{dt} = -4e^{-1000t} + 16e^{-4000t}$

$\dfrac{di}{dt} = 0$ when $16e^{-4000t} = 4e^{-1000t}$

or $e^{3000t} = 4$

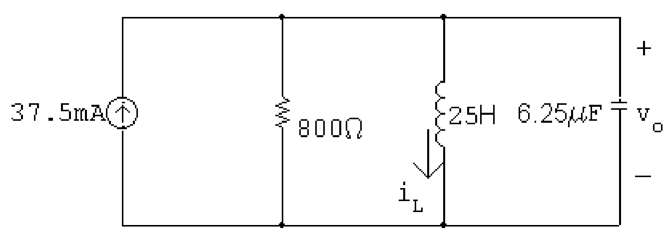
$\therefore \quad t = \dfrac{\ln 4}{3000}\mu\text{s} = 462.10\,\mu\text{s}$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 5\dfrac{di}{dt} = [-20e^{-1000t} + 80e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 6.24 $i_{\text{L}}(0^-) = i_{\text{L}}(0^+) = 37.5\,\text{mA}$

For $t > 0$



$i_{\text{L}}(0^-) = i_{\text{L}}(0^+) = 37.5\,\text{mA}$

$\alpha = \dfrac{1}{2RC} = 100\,\text{rad/s}; \qquad \omega_o^2 = \dfrac{1}{LC} = 6400$

$s_1 = -40\,\text{rad/s} \qquad s_2 = -160\,\text{rad/s}$

$v_o(\infty) = 0 = V_f$

$v_o = A_1' e^{-40t} + A_2' e^{-160t}$

$i_{\text{C}}(0^+) = -37.5 + 37.5 + 0 = 0$

$\therefore \quad \dfrac{dv_o}{dt} = 0$

$\dfrac{dv_o}{dt}(0) = -40A_1' - 160A_2'$

$\therefore \quad A_1' + 4A_2' = 0; \qquad A_1' + A_2' = 0$

$\therefore \quad A_1' = 0; \qquad A_2' = 0$

$\therefore \quad v_o = 0 \text{ for } t \geq 0$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

Hence the 37.5 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit the 30 V source sustains a current of 37.5 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 6.25 $\quad i_C(0) = 0; \qquad v_o(0) = 200\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{4}{2(0.04)} = 50\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^3}{0.4} = 2500$$

$$\therefore \quad \alpha^2 = \omega_o^2; \qquad \text{critical damping}$$

$$v_o(t) = V_f + D_1' te^{-50t} + D_2' e^{-50t}$$

$$V_f = 100\,\text{V}$$

$$v_o(0) = 100 + D_2' = 200; \qquad D_2' = 100\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -50D_2' + D_1' = 0$$

$$D_1' = 50D_2' = 5000 \text{ V/s}$$

$$v_o = 100 + 5000te^{-50t} + 100e^{-50t}\,\text{V}, \quad t \geq 0$$

P 6.26 $\quad \alpha = 800\,\text{rad/s}; \qquad \omega_d = 600\,\text{rad/s}$

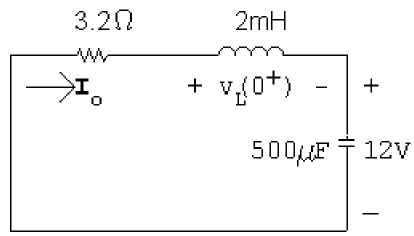
$$\omega_o^2 - \alpha^2 = 36 \times 10^4; \qquad \omega_o^2 = 100 \times 10^4; \quad w_o = 1000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 800; \qquad R = 1600L$$

$$\frac{1}{LC} = 100 \times 10^4; \qquad L = \frac{10^6}{(100 \times 10^4)(500)} = 2\,\text{mH}$$

$$\therefore \quad R = 3.2\,\Omega$$

$$i(0^+) = B_1 = 0\,\text{A}; \qquad \text{at } t = 0^+$$



$$12 + 0 + v_\text{L}(0^+) = 0; \qquad v_\text{L}(0^+) = -12\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-12}{0.002} = -6000\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 600B_2 - 800B_1 = -6000$$

$$\therefore \quad 600B_2 = 800B_1 - 6000; \qquad \therefore \quad B_2 = -10\,\text{A}$$

$$\therefore \quad i = -10e^{-800t}\sin 600t\,\text{A}, \quad t \geq 0$$

P 6.27 From Prob. 6.26 we know v_c will be of the form

$$v_c = B_3e^{-800t}\cos 600t + B_4e^{-800t}\sin 600t$$

From Prob. 6.26 we have

$$v_c(0) = -12\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_\text{C}(0)}{C} = 0$$

$$\frac{dv_c(0)}{dt} = 600B_4 - 800B_3$$

$$\therefore \quad 600B_4 = 800B_3 + 0; \qquad B_4 = -16\,\text{V}$$

$$v_c(t) = -12e^{-800t}\cos 600t - 16e^{-800t}\sin 600t\,\text{V} \qquad t \geq 0$$

P 6.28 $v_C(0^+) = \frac{1}{2}(240) = 120\,\text{V}$

$$i_L(0^+) = 60\,\text{mA}; \qquad i_L(\infty) = \frac{240}{5} \times 10^{-3} = 48\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2(2500)(5)} = 40$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{400} = 2500$$

$$\alpha^2 = 1600; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$$

$$s_{1,2} = -40 \pm j\sqrt{2500 - 1600} = -40 \pm j30 \text{ rad/s}$$

$$\begin{aligned} i_L &= I_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t \\ &= 48 + B_1' e^{-40t}\cos 30t + B_2' e^{-40t}\sin 30t \end{aligned}$$

$$i_L(0) = 48 + B_1'; \qquad B_1' = 60 - 48 = 12\,\text{mA}$$

$$\frac{di_L}{dt}(0) = 30B_2' - 40B_1' = \frac{120}{80} = 1.5 = 1500 \times 10^{-3}$$

$$\therefore \quad 30B_2 = 40(12) \times 10^{-3} + 1500 \times 10^{-3}; \qquad B_2' = 66\,\text{mA}$$

$$\therefore \quad i_L = 46 + 12e^{-40t}\cos 30t + 66e^{-40t}\sin 30t\,\text{mA}, \qquad t \geq 0$$

P 6.29 $\alpha = \frac{R}{2L} = 5000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{200} = 50 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 50 \times 10^6} = -5000 \pm j5000\,\text{rad/s}$$

$$v_o = V_f + B_1' e^{-5000t}\cos 5000t + B_2' e^{-5000t}\sin 5000t$$

$$v_o(0) = 0 = V_f + B_1'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore \quad B_1' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = 5000B_2' - 5000B_1'$$

$$\therefore \quad B_2' = B_1' = -40\,\text{V}$$

$$v_o = 40 - 40e^{-5000t}\cos 5000t - 40e^{-5000t}\sin 5000t\,\text{V}, \quad t \geq 0$$

P 6.30 $\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(1600)(6.25)} = 100; \qquad \alpha^2 = 10^4$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(25)(6.25)} = 6400$$

$$s_{1,2} = -200 \pm \sqrt{10^4 - 6400} = -100 \pm 60$$

$$s_1 = -40 \text{ rad/s}; \qquad s_2 = -160 \text{ rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$\therefore \quad v_o = A_1'e^{-40t} + A_2'e^{-160t}$$

$$v_o(0) = 30 = A_1' + A_2'$$

Note: $\qquad i_{\text{C}}(0^+) = 0$

$$\therefore \quad \frac{dv_o}{dt}(0) = 0 = -40A_1' - 160A_2'$$

Solving, $\qquad A_1' = 40\,\text{V}, \qquad A_2' = -10\,\text{V}$

$$v_o(t) = 40e^{-40t} - 10e^{-160t}\,\text{V}, \qquad t > 0^+$$

P 6.31 **[a]** $i_o = I_f + A_1'e^{-40t} + A_2'e^{-160t}$

$$I_f = \frac{30}{800} = 37.5\text{mA}; \qquad i_o(0) = 0$$

$$0 = 37.5 \times 10^{-3} + A_1' + A_2', \qquad \therefore \quad A_1' + A_2' = -37.5 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{30}{25} = -40A_1' - 160A_2'$$

Solving, $\qquad A_1' = -40\,\text{mA}; \qquad A_2' = 2.5\,\text{mA}$

$$i_o = 37.5 - 40e^{-40t} + 2.5e^{-160t}\,\text{mA}, \quad t \geq 0$$

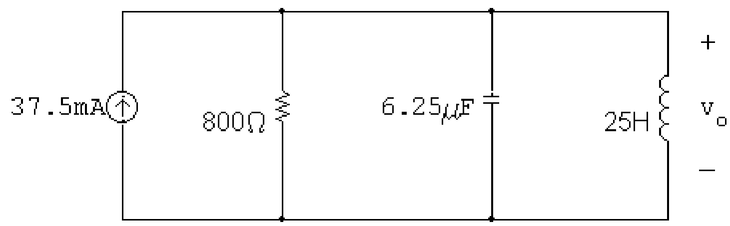
[b] $\dfrac{di_o}{dt} = [1600e^{-40t} - 400e^{-160t}] \times 10^{-3}$

$$L\frac{di_o}{dt} = 25(1.6)e^{-40t} - 25(0.4)e^{-160t}$$

$$\therefore \quad v_o = 40e^{-40t} - 10e^{-160t}\,\text{V}, \quad t \geq 0$$

This agrees with the solution to Problem 6.30.

P 6.32 For $t > 0$



$$\alpha = \frac{1}{2RC} = 100; \qquad \frac{1}{LC} = 6400$$

$$s_{1,2} = -100 \pm 60$$

$$s_1 = -40\,\text{rad/s}; \qquad s_2 = -160\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-40t} + A_2' e^{-160t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_C(0^+) = 37.5\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 6000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -40A_1' - 160A_2'$$

$$-40A_1' - 160A_2' = 6000$$

$$A_1' + 4A_2' = -150$$

$$A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 50\,\text{V}; \qquad A_2' = -50\,\text{V}$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

P 6.33 **[a]** From the solution to Prob. 6.32 $s_1 = -40\,\text{rad/s}$ and $s_2 = -160\,\text{rad/s}$, therefore

$$i_o = I_f + A_1' e^{-40t} + A_2' e^{-160t}$$

$$I_f = 37.5\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 37.5 + A_1' + A_2'; \qquad -40A_1' - 160A_2' = 0$$

It follows that

$$A_1' = -50\,\text{mA}; \qquad A_2' = 12.5\,\text{mA}$$

$$\therefore \quad i_o = 37.5 - 50e^{-40t} + 12.5e^{-160t}\,\text{mA}, \qquad t \geq 0$$

[b] $$\frac{di_o}{dt} = 2e^{-40t} - 2e^{-160t}$$

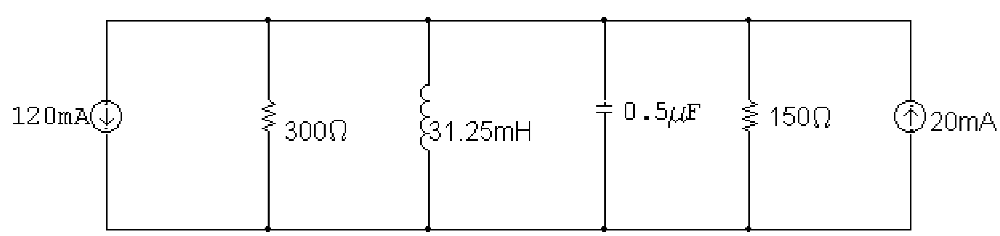
$$v_o = L\frac{di_o}{dt} = 25[2e^{-40t} - 2e^{-160t}]$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

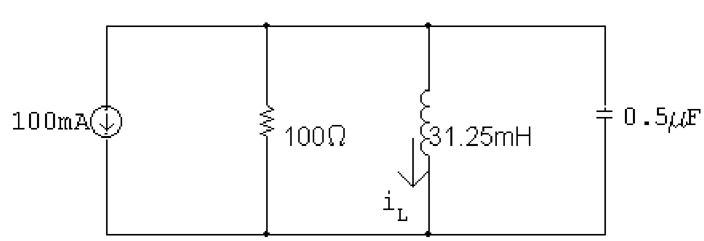
This agrees with the solution to Problem 6.32

P 6.34 $t < 0: \qquad i_L = 3/150 = 20\,\text{mA}$

$t > 0$:



$300\|150 = 100\,\Omega$



$$i_L(0) = 20\,\text{mA}, \qquad i_L(\infty) = -100\,\text{mA}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(31.25)(0.5)} = 64 \times 10^6; \qquad \omega_o = 8000\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(200)(0.5)} = 10^4; \qquad \alpha^2 = 100 \times 10^6$$

$$\alpha^2 - \omega_o^2 = (100 - 64)10^6 = 36 \times 10^6$$

$$s_{1,2} = -10{,}000 \pm 6000$$

$$s_1 = -4000 \text{ rad/s}; \qquad s_2 = -16{,}000 \text{ rad/s}$$

$$i_\text{L} = I_f + A_1' e^{-4000t} + A_2' e^{-16{,}000t}$$

$$i_\text{L}(\infty) = I_f = -100\text{mA}$$

$$i_\text{L}(0) = A_1' + A_2' + I_f = 20\,\text{mA}$$

$$\therefore \quad A_1' + A_2' - 100 = 20 \quad \text{so} \quad A_1' + A_2' = 120\,\text{mA}$$

$$\frac{di_\text{L}}{dt}(0) = 0 = -4000A_1 - 16{,}000A_2'$$

$$\text{Solving,} \qquad A_1' = 160\,\text{mA}, \qquad A_2' = -40\,\text{mA}$$

$$i_\text{L} = -100 + 160e^{-4000t} - 40e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$$

P 6.35 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{200} = 5000$

$$\alpha = \frac{R}{2L} = \frac{400}{40} = 10; \qquad \alpha^2 = 100$$

$$\alpha^2 < \omega_o^2 \quad \therefore \qquad \text{underdamped}$$

$$s_{1,2} = -10 \pm j\sqrt{4900} = -10 \pm j70 \text{ rad/s}$$

$$i = B_1 e^{-10t}\cos 70t + B_2 e^{-10t}\sin 70t$$

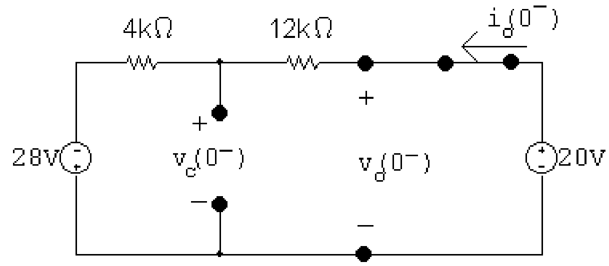
$$i(0) = B_1 = 147/420 = 350\,\text{mA}$$

$$\frac{di}{dt}(0) = 70B_2 - 10B_1 = 0$$

$$\therefore \quad B_2 = 50\,\text{mA}$$

$$i = 50e^{-10t}(7\cos 70t + \sin 70t)\,\text{mA}, \qquad t \geq 0^+$$

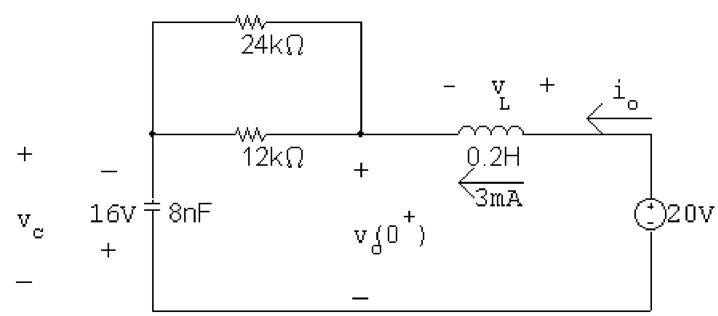
P 6.36 [a] $t < 0$:



$$i_o(0^-) = \frac{48}{16{,}000} = 3\,\text{mA}$$

$$v_C(0^-) = 20 - (12{,}000)(0.003) = -16\,\text{V}$$

$t = 0^+$:



$$12\,\text{k}\Omega \| 24\,\text{k}\Omega = 8\,\text{k}\Omega$$

$$\therefore \quad v_o(0^+) = (0.003)(8000) - 16 = 24 - 16 = 8\,\text{V}$$

and $v_L(0^+) = 20 - 8 = 12\,\text{V}$

[b] $v_o(t) = 8000 i_o + v_C$

$$\frac{dv_o}{dt}(t) = 8000\frac{di_o}{dt} + \frac{dv_C}{dt}$$

$$\frac{dv_o}{dt}(0^+) = 8000\frac{di_o}{dt}(0^+) + \frac{dv_C}{dt}(0^+)$$

$$20 = L\frac{di_o}{dt} + 8000 i_o + v_C$$

$$20 = 0.2\frac{di_o}{dt}(0^+) + 24 - 16$$

$$\therefore \quad 0.2\frac{di_o}{dt}(0^+) = 20 - 8 = 12$$

$$\frac{di_o}{dt}(0^+) = \frac{12}{0.2} = 60\ \text{A/s}$$

$$C\frac{dv_c}{dt}(0^+) = i_o(0^+)$$

$$\therefore \quad \frac{dv_c}{dt}(0^+) = \frac{3 \times 10^{-3}}{8 \times 10^{-9}} = 375{,}000$$

$$\therefore \quad \frac{dv_o}{dt}(0^+) = 8000(60) + 375{,}000 = 855{,}000 \text{ V/s}$$

[c] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{1.6} = 625 \times 10^6; \qquad \omega_o = 25{,}000 \text{ rad/s}$

$$\alpha = \frac{R}{2L} = \frac{8000}{0.4} = 20{,}000 \text{ rad/s}; \qquad \alpha^2 = 400 \times 10^6$$

$\alpha^2 < \omega_o^2$ underdamped

$$s_{1,2} = -20{,}000 \pm j15{,}000 \text{ rad/s}$$

$$v_o(t) = V_f + B_1' e^{-20{,}000t} \cos 15{,}000t + B_2' e^{-20{,}000t} \sin 15{,}000t$$

$$V_f = v_o(\infty) = 20 \text{ V}$$

$$8 = 20 + B_1'; \qquad B_1' = -12 \text{ V}$$

$$-20{,}000B_1' + 15{,}000B_2' = 855{,}000$$

Solving, $B_2' = 41 \text{ V}$

$$\therefore \quad v_o(t) = 20 - 12e^{-20{,}000t} \cos 15{,}000t + 41e^{-20{,}000t} \sin 15{,}000t \text{ V}, \qquad t \geq 0^+$$

P 6.37 **[a]** $t < 0$:

$$i_o = \frac{120}{8000} = 15 \text{ mA}; \qquad v_o = (5000)(0.015) = 75 \text{ V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{5000}{2(1)} = 2500 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1)(250)} = 4 \times 10^6 = 400 \times 10^4$$

$$\alpha^2 - \omega_o^2 = 625 \times 10^4 - 400 \times 10^4 = 225 \times 10^4$$

$$\therefore \quad s_{1,2} = -2500 \pm 1500$$

$$s_1 = -1000 \text{ rad/s} \qquad s_2 = -4000 \text{ rad/s}$$

$$\therefore \quad i_o(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$$

$$i_o(0) = A_1 + A_2 = 15 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = -1000A_1 - 4000A_2 = 0$$

Solving, $\quad A_1 = 20\,\text{mA}; \qquad A_2 = -5\,\text{mA}$

$$i_o(t) = 20e^{-1000t} - 5e^{-4000t}\,\text{mA}, \qquad t \geq 0^+$$

[b] $v_o(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$v_o(0) = A_1 + A_2 = 75$$

$$\frac{dv_o}{dt}(0) = -1000A_1 - 4000A_2 = \frac{-15 \times 10^{-3}}{250 \times 10^{-9}}$$

Solving, $\quad A_1 = 80\,\text{V}; \qquad A_2 = -5\,\text{V}$

$$v_o(t) = 80e^{-1000t} - 5e^{-4000t}\,\text{V}, \qquad t \geq 0^+$$

Check:

$$5000i_o + 1\frac{di_o}{dt} = v_o$$

$$5000i_o = 100e^{-1000t} - 25e^{-4000t}$$

$$\frac{di_o}{dt} = -20e^{-1000t} + 20e^{-4000t}$$

$$\therefore \quad 5000i_o + \frac{di_o}{dt} = 80e^{-1000t} - 5e^{-4000t}\,\text{V} \qquad \text{(checks)}$$

P 6.38 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{(20)(5)} = 10^4; \qquad \omega_o = 100 \text{ rad/s}$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(1600)(5)} = \frac{10^4}{80} = 125 \text{ rad/s}$$

$$s_{1,2} = -125 \pm \sqrt{(125)^2 - 10^4} = -125 \pm 75$$

$$s_1 = -50 \text{ rad/s}; \qquad s_2 = -200 \text{ rad/s}$$

$$I_f = 15\,\text{mA}$$

$$i_L = 15 + A_1' e^{-50t} + A_2' e^{-200t}$$

$$\therefore \quad -30 = 15 + A_1' + A_2'; \qquad A_1' + A_2' = -45 \times 10^{-3}$$

$$\frac{di_L}{dt} = -50A_1' - 200A_2' = \frac{60}{20} = 3$$

Solving, $\quad A_1' = -40\,\text{mA}; \qquad A_2' = -5\,\text{mA}$

$$i_L = 15 - 40e^{-50t} - 5e^{-200t}\,\text{mA}, \quad t \geq 0$$

P 6.39 $\quad \alpha = \frac{1}{2RC} = \frac{10^6}{(2500)(5)} = 80; \qquad \alpha^2 = 6400$

$$\omega_o^2 = 10^4$$

$$s_{1,2} = -80 \pm j\sqrt{10^4 - 6400} = -80 \pm j60 \text{ rad/s}$$

$$i_L = 15 + B_1' e^{-80t} \cos 60t + B_2' e^{-80t} \sin 60t$$

$$-30 = 15 + B_1' \qquad \therefore \quad B_1' = -45\,\text{mA}$$

$$\frac{di_L}{dt}(0) = -80B_1' + 60B_2' = 3$$

$$\therefore \quad B_2' = -10\,\text{mA}$$

$$i_L = 15 - 45e^{-80t} \cos 60t - 10e^{-80t} \sin 60t\,\text{mA}, \quad t \geq 0$$

P 6.40 $\quad \alpha = \frac{1}{2RC} = \frac{10^6}{(2000)(5)} = 100$

$$\alpha^2 = 10^4 = \omega_o^2 \qquad \text{critical damping}$$

$$i_L = I_f + D_1' te^{-100t} + D_2' e^{-100t} = 15 + D_1' te^{-100t} + D_2' e^{-100t}$$

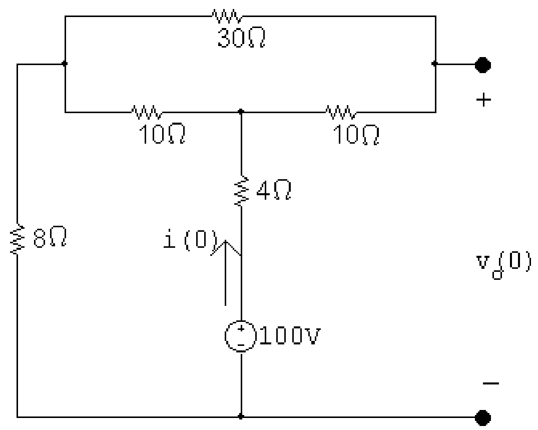
$$i_L(0) = -30 = 15 + D_2'; \qquad \therefore \quad D_2' = -45\,\text{mA}$$

$$\frac{di_L}{dt}(0) = -100D_2' + D_1' = 3000 \times 10^{-3}$$

$$\therefore \quad D_1' = 3000 \times 10^{-3} + 100(-45 \times 10^{-3}) = -1500 \times 10^{-3}$$

$$i_L = 15 - 1500te^{-100t} - 45e^{-100t}\,\text{mA}, \quad t \geq 0$$

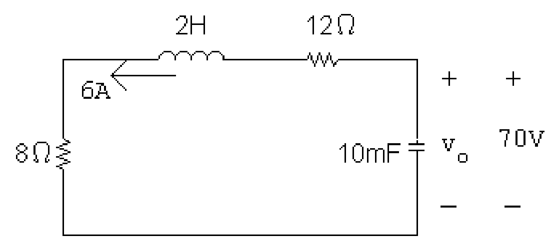
P 6.41 $t < 0$:



$$i(0) = \frac{100}{4+8+8} = \frac{100}{20} = 5\,\text{A}$$

$$v_o(0) = 100 - 5(4) - 10(5)\left(\frac{10}{50}\right) = 70\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{4} = 5, \qquad \alpha^2 = 25$$

$$\omega_o^2 = \frac{1}{LC} = \frac{100}{2} = 50$$

$\omega_o^2 > \alpha^2$ underdamped

$$v_o = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t; \qquad \omega_d = \sqrt{50-25} = 5$$

$$v_o = B_1 e^{-5t}\cos 5t + B_2 e^{-5t}\sin 5t$$

$$v_o(0) = B_1 = 70\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -5, \qquad \frac{dv_o}{dt} = \frac{-5}{10}\times 10^3 = -500\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -5B_1 + 5B_2 = -500$$

$$5B_2 = -500 + 5B_1 = -500 + 350; \qquad B_2 = -150/5 = -30\,\text{V}$$

$$\therefore \quad v_o = 70e^{-5t}\cos 5t - 30e^{-5t}\sin 5t\,\text{V}, \qquad t \geq 0$$

P 6.42 **[a]** Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\, e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \dfrac{\alpha}{\omega_d} B_1' = \dfrac{\alpha}{\omega_d}\dfrac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d} e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0^+$$

[b] $\dfrac{dv_o}{dt} = -\dfrac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\quad \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 6.43 **[a]** From Problem 6.42 we have

$$v_o = \frac{-V_g}{RC\omega_d} e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{120}{0.01} = 12{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{2500} = 400 \times 10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 16\,\text{krad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-600)10^9}{(120)(500)(16)\times 10^3} = 625$$

$$\therefore \quad v_o = 625e^{-12{,}000t}\sin 16{,}000t\,\text{V}$$

[b] From Problem 6.42

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{16{,}000}\tan^{-1}\left(\frac{16{,}000}{12{,}000}\right)$$

$$t_d = 57.96\,\mu\text{s}$$

[c] $v_{\max} = 625e^{-0.012(57.96)}\sin[(0.016)(57.96)] = 249.42\,\text{V}$

[d] $R = 12\,\Omega; \qquad \alpha = 1200\,\text{rad/s}$

$$\omega_d = 19{,}963.97\,\text{rad/s}$$

$$v_o = 5009.02e^{-1200t}\sin 19{,}963.97t\,\text{V}, \quad t \geq 0$$

$$t_d = 75.67\,\mu\text{s}$$

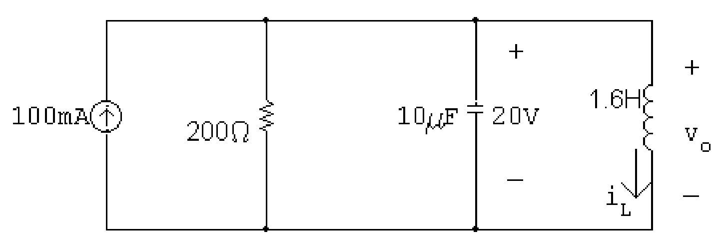
$$v_{\max} = 4565.96\,\text{V}$$

P 6.44 $t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{1000}{1250}(25) = 20\,\text{V}$$

$$i_{\text{L}}(0^-) = i_{\text{L}}(0^+) = 0$$

$t > 0$



$$-100 + \frac{20}{0.2} + i_{\text{C}}(0^+) + 0 = 0; \qquad \therefore \quad i_{\text{C}}(0^+) = 0$$

$$\frac{1}{2RC} = \frac{10^6}{(400)(10)} = 250\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{10(1.6)} = 62{,}500$$

$$\therefore \quad \alpha^2 = \omega_o^2 \quad \text{critically damped}$$

[a] $v_o = V_f + D_1' te^{-250t} + D_2' e^{-250t}$

$V_f = 0$

$$\frac{dv_o(0)}{dt} = -250D_2' + D_1' = 0$$

$v_o(0^+) = 20 = D_2'$

$D_1' = 250D_2' = 5000\,\text{V/s}$

$\therefore \quad v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}, \quad t \geq 0^+$

[b] $i_\text{L} = I_f + D_3' te^{-250t} + D_4' e^{-250t}$

$$i_\text{L}(0^+) = 0; \qquad I_f = 100\,\text{mA}; \qquad \frac{di_\text{L}(0^+)}{dt} = \frac{20}{1.6} = 12.5\ \text{A/s}$$

$\therefore \quad 0 = 100 + D_4'; \qquad D_4' = -100\,\text{mA};$

$-250D_4' + D_3' = 12.5; \qquad D_3' = -12.5\,\text{A/s}$

$\therefore \quad i_\text{L} = 100 - 12{,}500te^{-250t} - 100e^{-250t}\,\text{mA} \qquad t \geq 0$

P 6.45 **[a]** $w_\text{L} = \int_0^\infty p\,dt = \int_0^\infty v_o i_\text{L}\,dt$

$v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}$

$i_\text{L} = 0.1 - 12.5te^{-250t} - 0.1e^{-250t}\,\text{A}$

$p = 2e^{-250t} + 500te^{-250t} - 750te^{-500t} - 62{,}500t^2e^{-500t} - 2e^{-500t}\,\text{W}$

$$\frac{w_\text{L}}{2} = \int_0^\infty e^{-250t}\,dt + 250\int_0^\infty te^{-250t}\,dt - 375\int_0^\infty te^{-500t} -$$
$$31{,}250\int_0^\infty t^2e^{-500t}\,dt - \int_0^\infty e^{-500t}\,dt$$
$$= \frac{e^{-250t}}{-250}\Bigg|_0^\infty + \frac{250}{(250)^2}e^{-250t}(-250t-1)\Bigg|_0^\infty -$$
$$\frac{375}{(500)^2}e^{-500t}(-500t-1)\Bigg|_0^\infty -$$
$$\frac{31{,}250}{(-500)^3}e^{-500t}(500^2t^2 + 1000t + 2)\Bigg|_0^\infty -$$
$$\frac{e^{-500t}}{(-500)}\Bigg|_0^\infty$$

All the upper limits evaluate to zero hence

$$\frac{w_L}{2} = \frac{1}{250} + \frac{250}{62{,}500} - \frac{375}{25\times10^4} - \frac{(31{,}250)(2)}{(5)^3 10^6} - \frac{1}{500}$$

$$w_L = 8 + 8 - 3 - 1 - 4 = 8\,\text{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_L(\infty) = \frac{1}{2}(1.6)(0.1)^2 = 8\,\text{mJ}.$$

[b] $v = 5000te^{-250t} + 20e^{-250t}$ V

$$i_R = \frac{v}{200} = 25te^{-250t} + 0.1e^{-250t}\ \text{A}$$

$$p_R = vi_R = 2e^{-500t}[62{,}500t^2 + 500t + 1]$$

$$w_R = \int_0^\infty p_R\,dt$$

$$\frac{w_R}{2} = 62{,}500\int_0^\infty t^2e^{-500t}\,dt + 500\int_0^\infty te^{-500t}\,dt + \int_0^\infty e^{-500t}\,dt$$

$$= \frac{62{,}500e^{-500t}}{-125\times10^6}[25\times10^4t^2 + 1000t + 2]\Big|_0^\infty +$$

$$\frac{500e^{-500t}}{25\times10^4}(-500t-1)\Big|_0^\infty + \frac{e^{-500t}}{(-500)}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$\frac{w_R}{2} = \frac{62{,}500(2)}{125\times10^6} + \frac{500}{25\times10^4} + \frac{1}{500}$$

$$w_R = 2 + 4 + 4 = 10\,\text{mJ}$$

[c] $100 = i_R + i_C + i_L$ (mA)

$$i_R + i_L = 25{,}000te^{-250t} + 100e^{-250t} + 100$$

$$-12{,}500te^{-250t} - 100e^{-250t}\ \text{mA}$$

$$= 100 + 12{,}500te^{-250t}\ \text{mA}$$

$$\therefore\ i_C = 100 - (i_R + i_L) = -12{,}500te^{-250t}\ \text{mA} = -12.5te^{-250t}\ \text{A}$$

$$p_C = vi_C = [5000te^{-250t} + 20e^{-250t}][-12.5te^{-250t}]$$

$$= -250[250t^2e^{-500t} + te^{-500t}]$$

$$\frac{w_C}{-250} = 250\int_0^\infty t^2e^{-500t}\,dt + \int_0^\infty te^{-500t}\,dt$$

$$\frac{w_C}{-250} = \frac{250e^{-500t}}{-125 \times 10^6}[25 \times 10^4 t^2 + 1000t + 2]\Big|_0^\infty + \frac{e^{-500t}}{25 \times 10^4}(-500t - 1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-250(250)(2)}{125 \times 10^6} - \frac{250(1)}{25 \times 10^4} = -1000 \times 10^{-6} - 10 \times 10^{-4} = -2\,\text{mJ}$$

Note this 2 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_C(0) = \frac{1}{2}(10 \times 10^{-6})(20)^2 = 2\,\text{mJ}.$$

Thus $w_C(\infty) = 0\,\text{mJ}$ which agrees with the final value of $v = 0$.

[d] $i_s = 100\,\text{mA}$

$$p_s(\text{del}) = 100v_o\,\text{mW}$$
$$= 0.1[5000te^{-250t} + 20e^{-250t}]$$
$$= 2e^{-250t} + 500te^{-250t}\,\text{W}$$
$$\frac{w_s}{2} = \int_0^\infty e^{-250t}\,dt + \int_0^\infty 250te^{-250t}\,dt$$
$$= \frac{e^{-250t}}{-250}\Big|_0^\infty + \frac{250e^{-250t}}{62{,}500}(-250t - 1)\Big|_0^\infty$$
$$= \frac{1}{250} + \frac{1}{250}$$
$$w_s = \frac{2(2)}{250} = \frac{4}{250} = 16\,\text{mJ}$$

[e] $w_L = 8\,\text{mJ}$ (absorbed)

$w_R = 10\,\text{mJ}$ (absorbed)

$w_C = 2\,\text{mJ}$ (delivered)

$w_S = 16\,\text{mJ}$ (delivered)

$\sum w_{\text{del}} = w_{\text{abs}} = 18\,\text{mJ}.$

P 6.46 **[a]** $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(0.25)(160)} = \frac{10^8}{4} = 25 \times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\ \text{rad/s}$$

$$\therefore\quad R = (5000)(2)L = 2500\,\Omega$$

[b] $i(0) = i_L(0) = 24\,\text{mA}$

$$v_L(0) = 90 - (0.024)(2500) = 30\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{30}{0.25} = 120 \text{ A/s}$$

[c] $v_C = D_1 te^{-5000t} + D_2 e^{-5000t}$

$$v_C(0) = D_2 = 90\,\text{V}$$

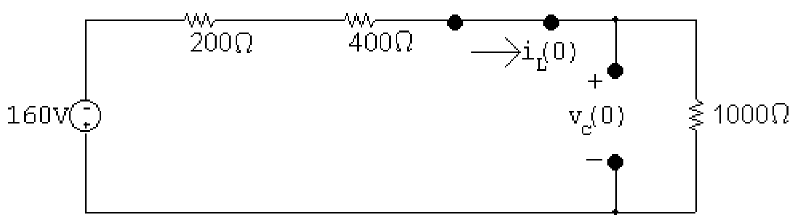
$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C}$$

$$D_1 - 450{,}000 = -\frac{24 \times 10^{-3}}{160 \times 10^{-9}} = -150{,}000$$

$$\therefore \quad D_1 = 300{,}000 \text{ V/s}$$

$$v_C = 300{,}000te^{-5000t} + 90e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

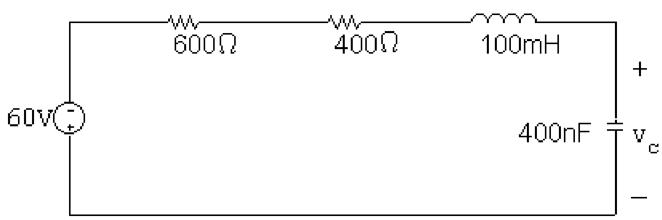
P 6.47 $t < 0$:



$$i_L(0) = \frac{-160}{1600} = -100\,\text{mA}$$

$$v_C(0) = 1000 i_L(0) = -100\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{1000}{200} \times 10^3 = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{(100)(400)} = \frac{10^8}{4} = 25 \times 10^6$$

$\omega_o = 5000$ rad/s $\quad\therefore\quad$ critical damping

$$v_C(t) = V_f + D_1' te^{-5000t} + D_2' e^{-5000t}$$

$$v_C(0) = -100\,\text{V}; \qquad V_f = -60\,\text{V}$$

$$\therefore\quad -100 = -60 + D_2'; \qquad D_2' = -40\,\text{V}$$

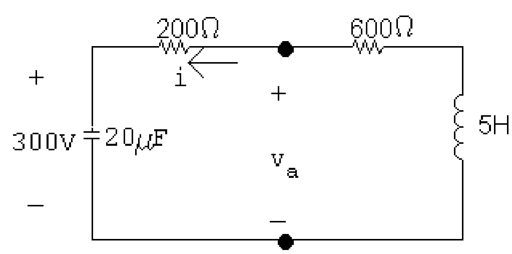
$$C\frac{dv_C}{dt}(0) = i_L(0) = -100 \times 10^{-3}$$

$$\frac{dv_C}{dt}(0) = \frac{-100 \times 10^{-3}}{400 \times 10^{-9}} = -250{,}000 \text{ V/s}$$

$$\therefore\quad D_1' = 5000(-40) - 250{,}000 = -450{,}000$$

$$v_C(t) = -60 - 450{,}000te^{-5000t} - 40e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 6.48 **[a]** For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = 300\,\text{V}$

[b] $v_a = 200i + 5 \times 10^4 \int_0^t i\,dx + 300$

$$\frac{dv_a}{dt} = 200\frac{di}{dt} + 5 \times 10^4 i$$

$$\frac{dv_a(0^+)}{dt} = 200\frac{di(0^+)}{dt} + 5 \times 10^4 i(0^+) = 200\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 300$$

$$\frac{di(0^+)}{dt} = -0.2(300) = -60\,\text{A/s}$$

$$\therefore\quad \frac{dv_a(0^+)}{dt} = -12{,}000\,\text{V/s}$$

[c] $\alpha = \dfrac{R}{2L} = \dfrac{800}{10} = 80\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(5)(20)} = 10^4$$

$$s_{1,2} = -80 \pm \sqrt{6400 - 10^4} = -80 \pm j60\,\text{rad/s}$$

Underdamped:

$$v_a = B_1 e^{-80t}\cos 60t + B_2 e^{-80t}\sin 60t$$

$$v_a(0) = B_1 = 300\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -80B_1 + 60B_2 = -12{,}000; \qquad \therefore\quad B_2 = 200$$

$$v_a = 300e^{-80t}\cos 60t + 200e^{-80t}\sin 60t\,\text{V}, \quad t \ge 0^+$$

P 6.49 **[a]** When $L = 1.6$ nH,

$$s_{1,2} = -\frac{100}{3.2\times 10^{-9}} \pm \sqrt{\left(\frac{100}{3.2}\times 10^9\right)^2 - \frac{10^{12}}{1.6\times 10^{-9}}}$$
$$= -3.125\times 10^{10} \pm 1.875\times 10^{10}$$

$$s_1 = -12.5\times 10^9\,\text{rad/s} \qquad s_2 = -50\times 10^9\,\text{rad/s}$$

$$\therefore\quad v_o = V_f + A_1' e^{-12.5\times 10^9 t} + A_2' e^{-50\times 10^9 t}$$

$$V_f = 5\text{V}$$

$$v_o(0) = 1\text{V} = A_1' + A_2' + 5$$

$$\frac{dv_o(0)}{dt} = 0 = -12.5\times 10^9 A_1' - 50\times 10^9 A_2'$$

$$\therefore\quad A_1' + A_2' = -4; \qquad A_1' = -4A_2'$$

$$\therefore\quad A_1' = -\frac{16}{3}\text{V}; \qquad A_2' = \frac{4}{3}\text{V}$$

$$\therefore\quad v_o = 5 - \frac{16}{3}e^{-12.5\times 10^9 t} + \frac{4}{3}e^{-50\times 10^9 t}\text{V} \qquad t \ge 0$$

[b] When $L = 2.5$ nH,

$$\frac{R}{2L} = 2\times 10^{10}; \qquad \left(\frac{R}{2L}\right)^2 = 4\times 10^{20}$$

$$\frac{1}{LC} = \frac{10^{12}}{2.5\times 10^{-9}} = 4\times 10^{20}$$

$$\therefore \quad \left(\frac{R}{2L}\right)^2 = \frac{1}{LC}; \qquad s_{1,2} = -2 \times 10^{10} \text{ rad/s}$$

$$\therefore \quad v_o = V_f + D_1' t e^{-2\times 10^{10} t} + D_2' e^{-2\times 10^{10} t}$$

$$V_f = 5\text{V}$$

$$v_o(0) = 5 + D_2' = 1; \qquad D_2' = -4\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = D_1' - 2 \times 10^{10} D_2'$$

$$\therefore \quad D_1' = -8 \times 10^{10} \text{ V/s}$$

$$\therefore \quad v_o = 5 - 8 \times 10^{10} t e^{-2\times 10^{10} t} - 4 e^{-2\times 10^{10} t}\text{V}, \quad t \geq 0$$

[c] When $L = 5$ nH,

$$\frac{R}{2L} = \frac{50}{5} \times 10^9 = 10^{10}$$

$$\frac{1}{LC} = \frac{10^{12} \times 10^9}{5} = 2 \times 10^{20}$$

$$s_{1,2} = -10^{10} \pm \sqrt{10^{20} - 2 \times 10^{20}} = -10^{10} \pm j10^{10} \text{ rad/s}$$

$$v_o = 5 + B_1' e^{-10^{10} t} \cos 10^{10} t + B_2' e^{10^{10} t} \sin 10^{10} t$$

$$v_o(0) = 5 + B_1' = 1; \qquad B_1' = -4\text{V}$$

$$\frac{dv_o(0)}{dt} = -10^{10} B_1' + 10^{10} B_2' = 0; \qquad B_1' = B_2' = -4\text{V}$$

$$v_o = 5 - 4 e^{-10^{10} t} (\cos 10^{10} t + \sin 10^{10} t)\text{V}, \quad t \geq 0$$

[d] When $L = 25$ nH,

$$\frac{R}{2L} = \frac{50}{25} \times 10^9 = 2 \times 10^9 \left(\frac{R}{2L}\right)^2 = 4 \times 10^{18}$$

$$\frac{1}{LC} = \frac{10^{12} \times 10^9}{25} = 40 \times 10^{18}$$

$$s_{1,2} = -2 \times 10^9 \pm j6 \times 10^9 \text{ rad/s}$$

$$v_o = 5 + B_1' e^{-2\times 10^9 t} \cos 6 \times 10^9 t + B_2' e^{-2\times 10^9 t} \sin 6 \times 10^9 t$$

$$v_o(0) = 1 = 5 + B_1'; \qquad B_1' = -4\text{V}$$

$$\frac{dv_o(0)}{dt} = -2 \times 10^9 B_1' + 6 \times 10^9 B_2' = 0; \qquad B_2' = -\frac{4}{3}\text{V}$$

$$v_o = 5 - 4 e^{-2\times 10^9 t} (\cos 6 \times 10^9 t + (1/3) \sin 6 \times 10^9 t)\text{V}, \quad t \geq 0$$

P 6.50 Use the $L = 0$ value of t_x as a first estimate. Then by successive approximations find that:

$t_x = 133.79$ ps when $L = 1.6$ nH

$t_x = 134.64$ ps when $L = 2.5$ nH

$t_x = 147.41$ ps when $L = 5$ nH

$t_x = 268.64$ ps when $L = 25$ nH