Matemática Discreta

Final - 2009.2

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1. $\{3,5\ pt\}$ Sejam a e b inteiros positivos. Prove que $ab = mdc(a,b) \cdot mmc(a,b)$. Dica: defina a e b como fatoração de primos e use as definições de mdc e mmc baseado em fatoração de primos. Caso precise, utilize o Teorema 1: $min(x,y) + max(x,y) = x + y$.

   Resposta:

   Sejam $a = p_1^{a_1} p_2^{a_2} \cdot \ldots \cdot p_n^{a_n}$ *e* $b = p_1^{b_1} p_2^{b_2} \cdot \ldots \cdot p_n^{b_n}$.

   $a \cdot b$
   $= (p_1^{a_1} p_2^{a_2} \cdot \ldots \cdot p_n^{a_n}) \cdot (p_1^{b_1} p_2^{b_2} \cdot \ldots \cdot p_n^{b_n})$ [*Definição a e b*]
   $= p_1^{a_1+b_1} p_2^{a_2+b_2} \cdot \ldots \cdot p_n^{a_n+b_n}$ [*Aritmética*]
   $= p_1^{min(a_1,b_1)+max(a_1,b_1)} \cdot \ldots \cdot p_n^{min(a_n,b_n)+max(a_n,b_n)}$ [*Teorema 1*]
   $= (p_1^{min(a_1,b_1)} \cdot \ldots \cdot p_n^{max(a_n,b_n)}) \cdot (p_1^{max(a_1,b_1)} \cdot \ldots \cdot p_n^{max(a_n,b_n)})$ [*Aritmética*]
   $= mdc(a,b) \cdot mmc(a,b)$ [*Def. mdc e mmc*]

2. $\{3,5\ pt\}$ Dadas as premissas $\forall x(P(x) \to (Q(x) \land S(x)))$ e $\forall x(P(x) \land R(x))$, prove que $\exists x(R(x) \land S(x))$.

   Resposta:

1. $\forall x(P(x) \to Q(x) \land S(x))$	[*Premissa*]
2. $P(c) \to Q(c) \land S(c)$	[*45 em 1*]
3. $\forall x(P(x) \land R(x))$	[*Premissa*]
4. $P(c) \land R(c)$	[*45 em 3*]
5. $P(c)$	[*42 em 4*]
6. $Q(c) \land S(c)$	[*37 em 5 e 2*]
7. $S(c)$	[*42 em 6*]
8. $R(c)$	[*42 em 4*]
9. $R(c) \land S(c)$	[*43 em 7 e 8*]
10. $\exists x(R(x) \land S(x))$	[*48 em 9*]

3. $\{3,0\ pt\}$ Calcule (na sua base preferida) e dê a resposta em *decimal*.

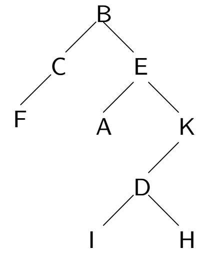
   a) $(10010011)_2 \cdot (101)_2$. **Resposta:** 735.

   b) $(C3)_{16} + (8D)_{16}$. **Resposta:** 336.

   c) $(1101010110)_2 + (101110100)_2$. **Resposta:** 1226.

4. $\{1,0\ pt\ EXTRA\}$ Descreva o caminho a ser percorrido na árvore abaixo usando os algoritmos pré-ordem, em-ordem e pós-ordem.

Resposta: pré-ordem: B-C-F-E-A-K-D-I-H
em-ordem: F-C-B-A-E-I-D-H-K
pós-ordem: F-C-A-I-H-D-K-E-B



$$\mathsf{T} \equiv \neg \mathsf{F} \tag{1}$$
$$\neg \mathsf{T} \equiv \mathsf{F} \tag{2}$$
$$p \wedge \mathsf{T} \equiv p \tag{3}$$
$$p \vee \mathsf{F} \equiv p \tag{4}$$
$$p \vee \mathsf{T} \equiv \mathsf{T} \tag{5}$$
$$p \wedge \mathsf{F} \equiv \mathsf{F} \tag{6}$$
$$p \vee p \equiv p \tag{7}$$
$$p \wedge p \equiv p \tag{8}$$
$$\neg(\neg p) \equiv p \tag{9}$$
$$p \vee q \equiv q \vee p \tag{10}$$
$$p \wedge q \equiv q \wedge p \tag{11}$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r) \tag{12}$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \tag{13}$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \tag{14}$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \tag{15}$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q \tag{16}$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q \tag{17}$$
$$p \vee (p \wedge q) \equiv p \tag{18}$$
$$p \wedge (p \vee q) \equiv p \tag{19}$$
$$p \vee \neg p \equiv \mathsf{T} \tag{20}$$
$$p \wedge \neg p \equiv \mathsf{F} \tag{21}$$
$$p \to q \equiv \neg p \vee q \tag{22}$$
$$p \to q \equiv \neg q \to \neg p \tag{23}$$
$$p \vee q \equiv \neg p \to q \tag{24}$$
$$p \wedge q \equiv \neg(p \to \neg q) \tag{25}$$
$$\neg(p \to q) \equiv p \wedge \neg q \tag{26}$$
$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r) \tag{27}$$
$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r \tag{28}$$
$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r) \tag{29}$$
$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r \tag{30}$$
$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p) \tag{31}$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q \tag{32}$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q) \tag{33}$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q \tag{34}$$
$$\neg \exists x P(x) \equiv \forall x \neg P(x) \tag{35}$$
$$\neg \forall x P(x) \equiv \exists x \neg P(x) \tag{36}$$
$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array} \tag{37}$$
$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array} \tag{38}$$
$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array} \tag{39}$$
$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array} \tag{40}$$
$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array} \tag{41}$$
$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array} \tag{42}$$
$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array} \tag{43}$$
$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array} \tag{44}$$
$$\begin{array}{l} \forall x P(x) \\ \hline \therefore P(c) \end{array} \tag{45}$$
$$\begin{array}{l} P(c), \textit{para um } c \textit{ arbitrário} \\ \hline \therefore \forall x P(x) \end{array} \tag{46}$$
$$\begin{array}{l} \exists x P(x) \\ \hline \therefore P(c), \textit{para algum } c \end{array} \tag{47}$$
$$\begin{array}{l} P(c), \textit{para algum } c \\ \hline \therefore \exists x P(x) \end{array} \tag{48}$$
$$a \notin A \equiv \neg(a \in A) \tag{49}$$
$$\{x \mid x \in A\} = A \tag{50}$$
$$P(a) \equiv a \in \{x \mid x \in A\} \tag{51}$$
$$(A = B) \equiv \forall x(x \in A \leftrightarrow x \in B) \tag{52}$$
$$(A \subseteq B) \equiv \forall x(x \in A \to x \in B) \tag{53}$$
$$\begin{aligned} (A \subset B) \quad \equiv \quad & \forall x(x \in A \to x \in B) \wedge \\ & \exists x(x \in B \wedge x \notin A) \end{aligned} \tag{54}$$
$$\emptyset \subseteq S, \textit{para todo } S \tag{55}$$
$$S \subseteq S, \textit{para todo } S \tag{56}$$
$$(A \times \emptyset) = (\emptyset \times A) = \emptyset \tag{57}$$
$$A \cup B = \{x \mid x \in A \ \vee \ x \in B\} \tag{58}$$
$$(x \in A \ \vee \ x \in B) \ \equiv \ (x \in (A \cup B)) \tag{59}$$
$$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\} \tag{60}$$
$$(x \in A \ \wedge \ x \in B) \ \equiv \ (x \in (A \cap B)) \tag{61}$$
$$|A \cup B| = |A| + |B| - |A \cap B| \tag{62}$$
$$A - B = \{x \mid x \in A \ \wedge \ x \notin B\} \tag{63}$$
$$(x \in A \ \wedge \ x \notin B) \ \equiv \ (x \in (A - B)) \tag{64}$$
$$\overline{A} = \{x \mid x \notin A\} \tag{65}$$
$$(x \notin A) \ \equiv \ (x \in \overline{A}) \tag{66}$$
$$A \cup \emptyset = A \tag{67}$$
$$A \cap U = A \tag{68}$$
$$A \cup U = U \tag{69}$$
$$A \cap \emptyset = \emptyset \tag{70}$$
$$A \cup A = A \tag{71}$$
$$A \cap A = A \tag{72}$$
$$\overline{(\overline{A})} = A \tag{73}$$
$$A \cup B = B \cup A \tag{74}$$
$$A \cap B = B \cap A \tag{75}$$
$$A \cup (B \cup C) = (A \cup B) \cup C \tag{76}$$
$$A \cap (B \cap C) = (A \cap B) \cap C \tag{77}$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \tag{78}$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \tag{79}$$
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \tag{80}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B} \tag{81}$$
$$A \cup (A \cap B) = A \tag{82}$$
$$A \cap (A \cup B) = A \tag{83}$$
$$A \cup \overline{A} = U \tag{84}$$
$$A \cap \overline{A} = \emptyset \tag{85}$$